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SCHOOL SCIENCE AND MATHEMATICS

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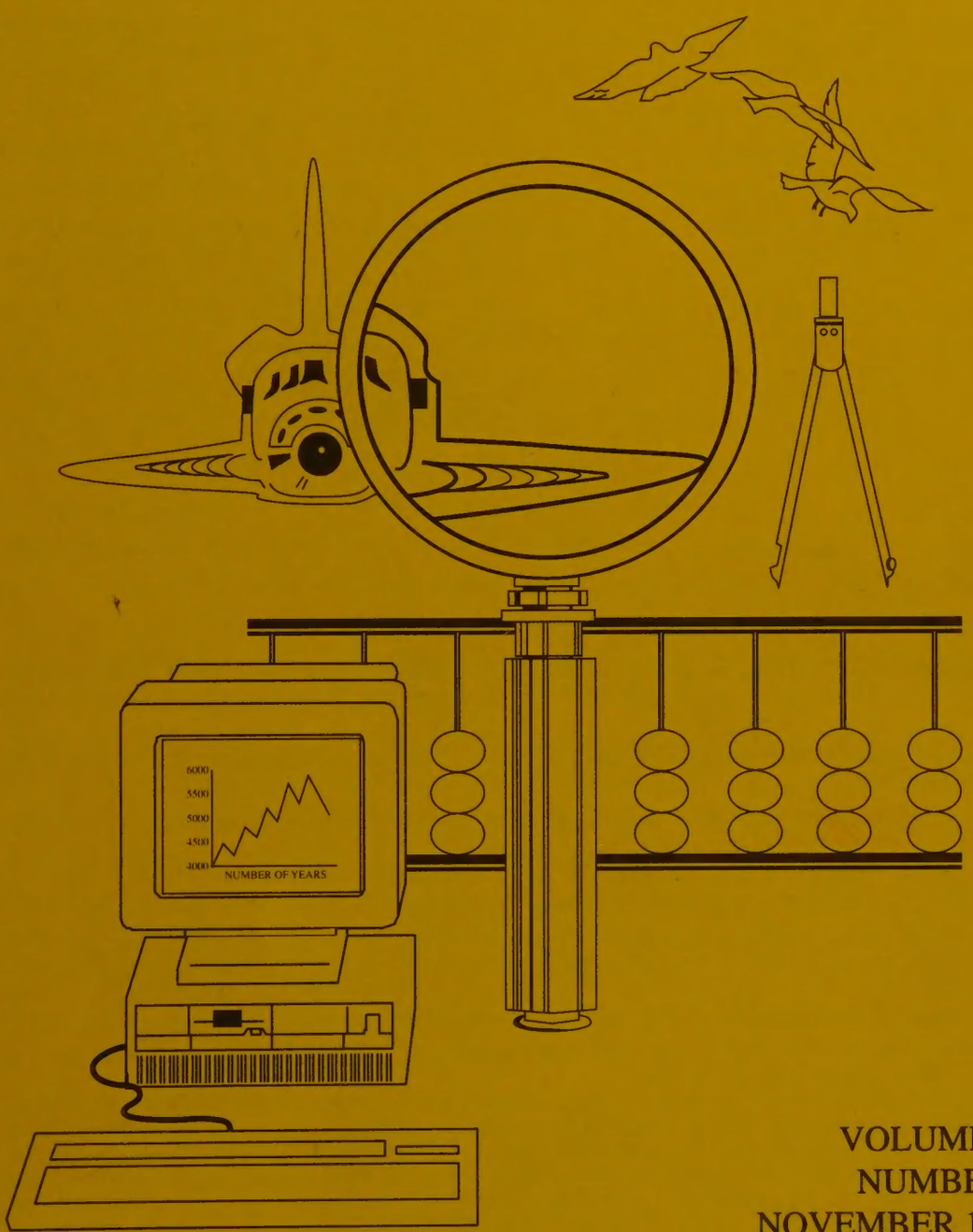

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EDITORIAL

Robert G. Underhill

NO18'94

The Need for Systemic Change

For over three decades the National Science Foundation has provided very large sums of money for improving mathematics and science learning for citizens of the United States. Much of this effort was triggered in 1957 by the launching of Sputnik by the U.S.S.R. NSF financed curriculum reform; academic year institutes; change models which focused on inclusion of parents, business and other constituencies; development of national standards; elaborate summer and academic year in-service programs; and so on. Yet, change has been frustratingly slow. National and international assessments have continued to reveal that United States learners can perform acceptably on low level tasks which require memory and limited comprehension. U.S. learners continue to perform very poorly on higher order thinking tasks requiring problem solving, reasoning, application and synthesis.

The problem has been adequately diagnosed: Changes in mathematics and science must be viewed within the context of a very complex system. There are school and university cultures; there are competing demands of traditional assessment practices and the curriculum standards represented by *Project 2061* and the *NCTM Standards;* there are competing expectations and needs of parents, teachers, business and industry, and the professional math and science communities; there are competing images of appropriate learning tools (e.g., textbooks, computers, calculators, science apparatus, and math manipulatives); there is tension between traditional didactic, lecture approaches and more recently developed collaborative learning models; there are tensions between traditional views of behavioral learning models and constructivist views of learning.

Underlying all of these tensions is a current belief or assumption that if we can work on all of these parts of the system at once, we can accelerate the rate of change. This is *systemic change*; this is the motivation behind NSF's State Systemic Initiatives (SSIs), Urban Systemic Initiatives (USIs) and Rural Systemic Initiatives (RSIs). NSF has provided funding to over 35 entities to operationalize the concept of systemic change.

The standard of living of vast numbers of United States citizens is on the line. The U.S.A. has many unskilled workers who cannot find jobs, and many jobs requiring mathematical, scientific and technological skills are going unfilled. There is a crisis brewing: Many of our citizens must lower their standard of living as they compete in the international marketplace for unskilled, low paying jobs *or* they must be educated in ways which will enhance their higher order thinking and technological skills. The gauntlet is down: Can we achieve consensus; can we change schools more quickly; can we maintain a high standard of living for our workers? The challenge must be accepted by *ALL*. This is not *their* problem; this is *our* problem. This is not *their* challenge; this is *OUR* challenge.

The Call

School Science and Mathematics readers are strongly encouraged to use this journal as a forum for presenting viewpoints and discussing problems and alternatives in the process of designing, implementing and achieving systemic change in timely and efficient ways. Please share your thoughts with our readers.

Integrated Science: The Importance of "How" Versus "What"

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Current problems in science education are identified from research literature. A review and analysis of major reform efforts in science education is provided. A summary of the Constructivist Learning Theory is presented; this theory has been generally accepted as a means for resolving current problems and for implementing the major reforms in science education. It is noted that specific constructivist teaching practices are needed to improve learning: Changes in teaching, not curriculum, are the key to effective reform. Many continue to think of reform in terms of content structure and curriculum frameworks, but to promote real learning, "how" to teach must be emphasized more than "what" to teach.

The Current Crisis In Science Education

Integrated science is in vogue again! Our current crisis in science education is stimulating new reform movements. This "current crisis" is seen by our political leaders as related to the economy. They have faith that our national well-being, including the ties to economic security, is related to achievement in science and technology. Because political leaders perceive a crisis, funding is available from the Department of Education (Eisenhower Act formerly, the Education for Economic Security Act) and the National Science Foundation (largely, Elementary, Secondary, and Informal Science Education Division) and a host of other federal departments and agencies (including the Departments of Agriculture and Energy and the Environmental Protection Agency). A coordinating group has been developed (Federal Coordinating Council for Science, Engineering and Technology) to assure that the problems in science, mathematics, and technology education are addressed (in coordinated and systematic ways) with national funding.

These initiatives stress the value of science in promoting a stronger economy, a better work force, and higher test scores (to enable the United States to be first in the world in science and mathematics by the year 2000). Yet, the science education community identifies different (and perhaps more fundamental) problems in science education:

1. Although a focus on science process skills (inquiry, critical thinking, problem solving) has been a goal of science education reformers for over fifty years (NSSE, 1932; Welch, 1981), there is virtually no evidence that typical science courses have been successful in stimulating growth across grade levels in student mastery of such skills (Hurd, 1978; Welch, 1981). This is particularly alarming in view of the supposedly great attention to science-as-inquiry during the past thirty years in NSF-supported projects.

2. Students in science classes are more interested in and more knowledgeable about technology than about basic science concepts and processes (Miller, Suchner & Voelker, 1980; Voelker, 1982). This may come as a surprise: technology was consistently deleted from K-12 science curricula during the two decades after the Soviet launching of Sputnik (Harms & Yager, 1981). Yet technology provides a tie to current problems and can provide the link between science and the daily lives of real people.

3. Although curiosity is the starting place for science (and technology), typical school science programs destroy natural curiosity. The longer students study science, the less curious they are about the natural world. Students in lower elementary school are very curious; by the senior year of high school curious students are rare; even those who voluntarily enroll in advanced placement courses rarely demonstrate curiosity. Success of such science students is too often measured by their ability to repeat what they hear in lecture, to verify "experimental" results in the laboratory, and to produce calculations and explanations for the purpose of satisfying their teachers (Hueftle, Rakow & Welch, 1983; NAEP, 1978, 1988).

4. Dealing with cause and effect relationships is basic to the scientific enterprise, but school science instruction tends to diminish the ability of students to understand such relationships (Helgeson, Blosser & Howe, 1977; NAEP, 1978). Successful students learn to mimic what teachers think scientists do and know. Unfortunately, one rarely finds a student who

can apply science knowledge elsewhere after mastering concepts and process skills in science classrooms (Mestre & Lochhead, 1990; Weiss, 1978, 1987). Students who appear to know basic science concepts and process skills, but who cannot use those concepts and skills, provide evidence of a serious instructional problem (Champagne & Klopfer, 1984; Mestre & Lochhead, 1990).

5. Nearly all students perceive school science as unrelated to their daily lives. They see few ties between the science they study and personal concerns such as their diets, health, hobbies, work experiences, and consumer decisions. The highest achieving students see science classes primarily as preparation for future, more advanced courses (Harms, 1977; NAEP, 1978, 1988; Yager & Penick, 1986).

6. Most students elect to halt study of science as soon as they are permitted to do so (Harms & Yager, 1981; Weiss, 1978, 1987). (Merely requiring more years of the same kind of study, a corrective stratagem which has been tried in many states, does not produce more scientifically technologically literate graduates!) Students report an intensified lack of interest in science careers, science study, science classes, and more negative views of science teachers the longer they are enrolled in school (NAEP, 1978, 1988; Yager & Penick, 1986). The evidence documents failure of the current situation.

7. Most students view science as unrelated to potential careers (Harms & Yager, 1981; Weiss, 1987). Few see science courses as places to learn anything that will be valuable when earning a living.

8. Newspaper reports and summaries of special studies have provided ample evidence of common misconceptions that students have of the natural world. These misconceptions are common to most students, even those enrolled in advanced college preparatory courses (Champagne & Klopfer, 1984; Mestre & Lochhead, 1990).

New Reform Efforts

To alleviate the current problems—those identified by both our political leaders and by science educators—three major reform efforts are underway nationally. These efforts are: (a) Project 2061, a project of the American Association for the Advancement of Science (AAAS), funded by the Carnegie and Mellon Foundations and the National Science Foundation (NSF); (b) Scope, Sequence, and Coordination (SS&C), a project of the National Science Teachers Association (NSTA), funded by the Department of Education, NSF, and American Petroleum Institute; and (c) Science/Technology/Society (STS), a project conceived by the National Association for Science, Technology and Society (NASTS) headquartered at Pennsylvania State University, funded by the National Science Foundation and a variety of other organizations and foundations.

These reforms are influenced by (or perhaps are influencing) a new major effort initiated by the government to establish national standards for science education. These standards are intended to parallel the national mathematics standards developed and promulgated by the National Council of Teachers of Mathematics. NSF funded the National Research Council of the National Academy of Science to administer the National Standards effort in science (National Committee on Science Education Standards and Assessment, 1992, 1993a, 1993b).

Project 2061

Project 2061 began with panels of scientists meeting to determine what every high school graduate should know with respect to biology (Clark, 1989), chemistry and physics (Bugliarello, 1989), mathematics (Blackwell & Henkin, 1989), social science (Appleby & Maher, 1989), and technology (Johnson, 1989). Later, a publication establishing a philosophical base for the project was produced (Rutherford & Ahlgren, 1989). This was designed for use in six school districts selected to develop and implement new science programs as models of 2061 reform.

These six districts are: San Francisco, San Diego, McFarland (WI), Philadelphia, San Antonio, and Athens (GA). After a few years of effort, 2061 has produced a 252-page document called Benchmarks for Science Literacy (AAAS, 1993) which identifies science, technology, and mathematics concepts considered appropriate at various grade levels across the K-12 continuum. The Benchmarks are meant to assist the six districts and to demonstrate to the funding groups that there is a serious attempt to focus upon a coordinated and articulated curriculum across grade levels.

Scope, Sequence, and Coordination

The NSTA Scope, Sequence, and Coordination (SS&C) project was conceived as an attempt to

coordinate all sciences (biology, chemistry, physics, and earth science) into a middle and high school sequence that would replace the "layer cake" curriculum characterizing most schools. Even junior high schools often have discipline-based courses similar to the biology, chemistry, and physics sequence that remains typical of grades 10 through 12. SS&C introduces basic science concepts, (for example, Newton's Second Law) that can be viewed in many discipline contexts, and reintroduces these same concepts both within a given year and across years; this is in recognition of the spacing effect (NSTA, 1992).

Another basic tenet of SS&C is to develop topics from concrete to abstract over 6-12 years. Terminology is introduced after meaning has been established. SS&C was funded to operate in Alaska, California, Iowa, North Carolina, Puerto Rico, and Texas. Its primary focus during 1990-1994 was to restructure science education at the middle school level.

Science/Technology/Society (STS)

The Science/Technology/Society (STS) movement began in the United States with Norris Harms' identification of STS as a primary focus for school science (along with biology, physical science, elementary science, and science as inquiry) with his NSF-supported Project Synthesis (Harms, 1977).

Later, the STS program at Pennsylvania State University assumed leadership and encouraged STS in schools. A network system was established to foster in-state and between state communications. These activities were supported via two major NSF grants, and resulted in the creation of a new "sponsoring" organization—the National Association for Science-Technology Society (NASTS). The original STS grant was called S-STS (Science through STS). It sought to collect and disseminate curriculum material from all over the world (especially from Europe, where the movement had flourished for ten years). S-STS also developed model curriculum materials to fill gaps in areas determined to be important as new reform programs were implemented.

The STS movement has been embraced by many environmental education leaders who sought both a place for specialized material and a broader framework to accomplish their goals. The social sciences also embrace STS both as a curriculum focus and as an approach to teaching and learning. The Social Science Consortium produced and distributed materials on STS; many were called "Lessons" (Hickman, Patrick & Bybee, 1987; LaRue, 1988; Patrick & Remy, 1985).

The Role Of Curriculum

All three science education reforms of the '90s have emphasized curricular materials. New curriculum has been seen as a means for meeting objectives; thus, new curriculum is believed to exemplify reform. Some of this emphasis on curriculum has been dictated by funding groups as well as educational and political leaders who advocate reform. "Reform" seems to be synonymous with improved materials. Reform efforts during the post-Sputnik era had been similarly focused. The NSF funding unit of the '60s was called the Course Content Improvement Section. Three and four decades ago reformers openly talked of producing teacher-proof curricula. Materials illustrated and indicated the reform.

The Importance Of "How" versus "What"

As efforts in the '90s continue (2061, SS&C, and STS) it is appropriate to question the importance of "what" is produced to initiate and sustain educational reform. Perhaps reform remains elusive because, contrary to typical emphasis, "how" teaching is practiced is more important than "what" is being used to accomplish reform.

Science for All Americans (AAAS, 1989) had greater appeal before the concepts in the *Benchmarks* document were identified and listed. The tenets (although always discussed in terms of fundamental concepts) may have had more power before specific conceptual frameworks and resulting materials were identified. Perhaps STS has greater potential when it is not circumscribed by modules, lessons, and materials. The current reforms all agree on certain parameters. These include:

Less is more. It is time for students to stop memorizing the names of all the molecules involved in photosynthesis. One study estimated that students encounter more new words in a high school biology book than in two years of instruction in a foreign language.

Teacher power. Post-Sputnik reforms sometimes took the tone of academic highbrows telling teachers what to do, and teachers have taken much of the blame for education's ills. But the new goal is to boost teachers, not bash them, giving them time to learn science themselves, rather than berating them for deficient backgrounds.

Science without walls. For example, sixth graders can be found who do not enroll in science,

math, social studies, or reading. Instead they take one big course 200 minutes run by four teachers that incorporates all four subjects. Real life problems rarely come labeled as "chemistry" or "math." How far should such course integration go? Different reform projects answer this question in diverse ways.

Do it yourself. Teachers are being trained to use an elementary science kit, doing all the experiments their students will do. At the end of the school day teachers may gather to learn how to separate salt from pepper using static electricity. A staple of the 1960s reforms, hands-on activities are back. They are improved, with a new emphasis on quality—"minds-on" as well as hands-on. Teachers are being trained more carefully and given the logistic support they need.

Two-way traffic. Students have not understood much of what they have been told, in part because no one has listened to their preexisting ideas. For example, research shows that many youngsters do not understand the concept of temperature. They think putting on a sweater keeps you warm because sweaters contain warmth. In order to elicit and change such ideas, teachers must recognize that they need to listen to the students as much as the students listen to them.

Science for everyone. It is vital to widen the science career pipeline, especially to include more women and minorities. Reformers now also recognize a second goal: producing scientifically literate citizens. Even students who will never become scientists need to understand more about how the world works. New programs are aimed at all students, not just the gifted.

Matchmaker, matchmaker. Scientists and teachers come from very different worlds. Elementary teachers often opted out of science as soon as they could, but they know their business, namely, students. Meanwhile many scientists would like to help with science education, but do not know how to do it (Culotta, 1990).

The "How" Of Learning

All three reform movements, as well as the committees at work on the NRC Science Standards, acknowledge the reports of cognitive scientists. The study of how human learning takes place was first funded in 1983 as a precursor to massive teacher enhancement and materials development activities. The premise was that our economic security would be strengthened by improving science, mathematics, and technology education in U.S. schools, and that this could best be accomplished by matching teaching techniques to learning behavior.

In an effort to elucidate the mechanism of brain functioning during learning, cognitive scientists (learning psychologists) sought out humans whose brains were presumably functioning well. After some debate, experiments were conducted utilizing, first, undergraduate physics majors and, later, engineering majors. It was assumed that these persons were the most academically successful: easily passing courses, performing well in classrooms and laboratories, and obviously interested in pursuing careers as physicists and engineers. The goal was to ascertain how their brains operated in order to use the information in attempts to get other brains, those of less able and less interested students, to function equally well.

The students were given real-world problems to solve. Usually, one student worked with one experimenter. The problems were difficult, perhaps even tricky, but that was the point: slow down the thinking/analyzing process to determine the steps involved. The results were astounding!

Instead of finding how successful minds worked, investigators found that they failed (Champagne & Klopfer, 1984; Driver, 1990; Driver & Bell, 1986; Mestre & Lochhead, 1990). The university physics majors (85%) and engineering majors (90%) could not solve real-world problems. There was obviously a vast difference between successful performance in classrooms and laboratories (where answers were given and most laboratories were verification-type) and application of knowledge to real-world problems. Apparently, the best science students, that is, the ones performing well and enjoying it, are little more than actors and actresses. They learn their lines well and act them out with feeling. This evokes praise and high grades from instructors. And yet, very few have mastered true learning. Learning ought to include demonstration of the ability to apply the knowledge gained. These students lacked application skills.

Cognitive science researchers probed further with the students, in an effort to find the reason for the inability to apply knowledge. What were the blocks to more accurate interpretation and performance when dealing with real-world problems? The researchers found that frequently the difficulty was a previous interpretation that each student had constructed for him or herself to interpret some object or event. These personally constructed (and erroneous) explanations were more powerful than all the

concepts, process skills, and directed experiences that school programs had provided. Apparently, when it comes time to apply knowledge to problems in the real world, even the "best" students ignore their science content knowledge and revert to a reliance on personal constructions.

Warnings from the past have had little impact on our massive reform efforts of the 35 years following Sputnik and/or our current efforts with 2061, SS&C, or STS. Recall Feynman's analysis of his visit in classrooms throughout Brazil (1985); or the logic of The Saber-tooth Curriculum (Benjamin, 1939); or review some of the writings of John Dewey (1938). We remain enamored of materials that define the goals, the curricula, the form of instruction, and the information we use to assess our successes. Yet the research seems clear: how we teach is more important than what we teach. Why do we continue to focus on the "what," while making the assumption that no real attention need be paid to the "how?"

The Role Of Learning Theory

The Constructivist Learning Theory offers important insights. If real reform is to emerge from the '90s (von Glasersfeld, 1987, 1988; Yager, 1991), efforts must focus on what we have learned from current research on human learning processes. Constructivist learning theory explains that real learning can only occur when individuals engage their minds in a process of actively constructing meaning for themselves. Real meaning cannot be transferred passively from teachers and/or textbooks to the minds of students. What is transferred is the ability to mimic—similar to the talent a parrot displays when it duplicates sounds made by a human.

Constructivism implies that students must be engaged in experiences that challenge their initial explanations and interpretations. Ideally, a classroom is a place where students' prior experiences, explanations, and ideas are elicited. These present opportunities for challenges, debates, and the designing of tests to determine the validity of explanations suggested. Experiences and explanations offered by professional scientists and engineers should be used as corroborating evidence, and/or challenged.

Learning is, by definition, an active process; the learner must be deliberately engaged. Unfortunately, however, the typical classroom/laboratory is a place where a student is taught to memorize, to repeat, to verify, or to develop rote skills for "future" use. Information and skills taught are offered as important for the future well-being of students, rather than for present use and value. Such skills and information are rarely seen as valuable, once instruction is over.

National Standards For Science Teaching

The current NRC effort to establish National Standards for Science Teaching has returned us to the debate of "what." Most of the debate so far has focused upon basic concepts of science. There has been an attempt to reduce the list of concepts and to leave determination of their relative importance to states and schools. It is clear that the resulting shorter list of big ideas in the disciplines is the most important facet of standards.

Establishing standards for teaching about the Nature of Science is seen as of secondary importance. Lists of the processes used by scientists are easily agreed upon. However, past efforts aimed at producing student learning and growth in this arena have been ineffective. Hurd (1978) was led to the following observation some 15 years ago:

> "The development of Enquiry skills as a major goal of instruction in science appears to have had only a minimal effect on secondary school teaching. The rhetoric about enquiry and process teaching greatly exceeds both the research on the subject and the classroom practice. The validity of the enquiry goal itself could profit from more scholarly interchange and confrontation even if it is simply to recognize that science is not totally confined to logical processes and data-gathering." (p. 62)

The Standards group has elected "Applications of Science" as its third category for developing standards. This, too, is a non controversial consideration. The 2061 effort has endorsed technology; SS&C has included "application" as the ultimate in achieving learning of science and science processes; STS includes technology as its unifying link between pure science and the society which produces and uses both science and technology.

The NRC Standards team has included Contexts of Science as its fourth and final consideration. Unfortunately, however, no work has been done in this arena. It remains as a fourth and, therefore, final consideration. Concepts remain first order, process skills second, and the necessity to move students to the applications level remains as third. It remains to

be seen what the NRC team will do with "Contexts."

The constructivist perspective for learning suggests that context is the most important aspect for determining whether or not learning will occur. For many students, the context for science is the place to start; context can provide the focus on "how" that has been so elusive with all past reform efforts. Considering context implies a focus on student prior experiences. This, in turn, will provide the basis for encouraging new experiences which may affect and/or alter previous conceptions.

Starting with concepts promotes continuation of the focus on "what." The work of cognitive scientists and constructivist researchers should force a new viewpoint from which to look at reform. Such analysis will inevitably result in sharper focus on how to teach, and relegate content to a lower level of importance.

Past As Prologue

Leaders in educational reform are just beginning to hear the message written out by children's authors more than a century ago. In her famous book Heidi (1956), first published in 1880, Johanna Spyri describes how Heidi is unable to learn to read due to her misconceptions. She is easily able to learn to read after she goes to visit Clara, and Clara's wise Grandmamma elicits and addresses Heidi's misconceptions about reading.

Children's literature advocated STS teaching as early as 1917, when Dorothy Canfield Fisher described a young girl's experiences living with cousins in Vermont. Betsy is helping to make butter: "She weighed out the salt needed on the scales, and was very much surprised to find that there really is such a thing as an ounce. She had never met it before outside the pages of her arithmetic book and she didn't know it lived anywhere else" (Understood Betsy, 1991, p. 52). Up until that moment, not only had eight-year-old Betsy never met a real ounce, she also had thought that all butter came from the store. She did not even know what it was made from. Fisher's book expounds a number of STS principles, including interdisciplinary instruction and cooperative group work in school.

Lucy Maud Montgomery, in both her famous *Anne Of Green Gables* series (1984) and her *Emily Of New Moon* (1983) books, describes the most effective teachers as those who use hands on activities that are personally relevant to students' daily lives. For example, geography is studied by having "school and playground mapped out into countries and you dressed up as the animals inhabiting them or traded in various commodities over their rivers and cities" (Montgomery, 1983, p. 294). This is from a book written in 1923!

Redesigning The Typical Classroom

Messages from the past, combined with what current research findings suggest, indicate the necessity of altering typical classrooms. The main emphasis should be on how to teach, not what to teach. Typical teaching involves first deciding on the concepts and skills that are to be taught, and then proceeding to teach in a direct manner, as though knowledge can be transferred intact from teacher to student. Constructivist teaching demands different approaches, because it recognizes that what students see and hear will be filtered through and affected by their personal beliefs. A constructivist teacher will:

- Encourage and accept student autonomy, initiation, and leadership.
- Allow student thinking to drive lessons: shift content and instructional strategy based on student responses.
- Ask students to elaborate on their responses.
- Allow wait time when asking questions.
- Encourage students to interact with each other and with teachers.
- Ask thoughtful, open-ended questions.
- Encourage students to reflect on experiences and predict future outcomes.
- Ask students to articulate their theories about concepts before the teacher presents his/her understanding of the concepts.
- Look for students' alternative conceptions and de sign lessons to address any misconceptions.

Project 2061 has done little (at least available to the outside world) to address "how" science should be taught. Some ideas are latent in *Science for All Americans*, but unfortunately, the *Benchmarks* document returns our attention to a hierarchy of concepts. SS&C was conceived as a new way of structuring the presentation of concepts. The assumption was that teachers would continue to direct and disseminate better (and less) material, carefully coordinated among the disciplines. STS is often defined as new content that focuses upon environmental, health, and ethical issues. But with a focus on added material, lessons, and vignettes, current

reform may inevitably suffer from the same problem as previous reforms: a "what" focus.

When the STS movement began in the United States early in the 1980s, NSTA recognized its importance and potential. A special task force was appointed to study the STS reform, and to define STS teaching strategies for the professional science education community. In 1990, the NSTA Board unanimously adopted the Task Force recommendations. The NSTA Board defined STS as: the teaching and learning of science and technology in the context of human experience (NSTA, 1990-1991, p. 47). Thus, NSTA has removed STS teaching from the "what" debate. STS answers the question of "how" to teach, not "what" to teach. Of course, there must be materials to use when teaching in ways designed to promote learning. Such materials are defined by their value for teaching in a constructivist manner and for stimulating real learning.

The NSTA definition of STS highlights the focus on context and its power for stimulating real learning. Context can make problems considered in school science synonymous with real world problems. When school science classes focus on realñworld problems, there is a good chance the cognitive scientists will find that successful students in school science and technology are also successful in solving problems in the real world, outside of school.

Constructivist teaching practices yield acceptable levels of student understanding of basic concepts (Iskandar, 19991; Liu, 1992; Lu, 1993; Mackinnu, 1991; Yager, Liu & Blunck, 1993; Yager, Liu & Varrella, 1993). Additionally, students from constructivist classrooms excel in the process domain. Skills they learn in real-world settings (contexts) are easily transferred to general use in new settings. In addition, students ask more and better questions (i.e., they are more curious after instruction; they are better at suggesting possible explanations and predicting consequences). Students who work and learn in constructivist settings are able to apply concepts and skills in all kinds of situations in and out of school. Their understanding of the nature of science is superior. Constructivist teaching practices and learning environments result in students with more positive attitudes about science classes, teachers, and courses.

Most reform efforts in the 1990s acknowledge the desirability of removing the rigid limits established to define the science disciplines per se. Thus, there may be less reason for (but more interest in) establishing maps of science concepts that can be used for purposes of integration or coordination. Any focus on reorganization of content may lead to failure to achieve an improvement in student learning. To improve student learning, it will be necessary to focus on a change in teaching. Teaching strategies must be congruent with learning processes: This cannot be said too often.

Appropriate teaching is more important than content. No matter what is on the list of critical content, constructivist research suggests that students will not learn unless teaching practices are designed to promote learning. *How we teach* is more vital than *what we teach*!

References

American Association for the Advancement of Science. (1989). *Science for all Americans: A Project 2061 report on literacy goals in science, mathematics, and technology.* Washington, DC: Author.

American Association for the Advancement of Science. (1993). *Benchmarks for science literacy: Part I: Achieving science literacy: Project 2061.* Washington, DC: Author.

Appleby, M. H., & Maher, W. B. (1989). *Social and behavioral sciences: Report of the Project 2061 phase I social and behavioral sciences panel.* Washington, DC: American Association for the Advancement of Science.

Benjamin, H. R. W. (1939). *The saber- tooth curriculum..* New York, NY: McGraw Hill Book Company.

Blackwell, D., & Henkin, L. (1989). *Mathematics: Report of the Project 2061 phase I mathematics panel.* Washington, DC: American Association for the Advancement of Science.

Bugliarello, G. (1989). *Physical and information sciences and engineering: Report of the Project 2061 phase I physical and information sciences and engineering panel.* Washington, DC: American Association for the Advancement of Science.

Champagne, A. B., & Klopfer, L. E. (1984). Research in science education: The cognitive psychology perspective. In D. Holdzkom & P. B. Lutz, (Eds.), *Research within reach: Science education* (pp. 171-189). Charleston, WV: Research and Development Interpretation Service, Appalachia Educational Laboratory.

Clark, M. (1989). *Biological and health sciences: Report of the Project 1061 phase I biological and health sciences panel.* Washington, DC: Ameri-

can Association for the Advancement of Science.

Culotta, E. (1990). Can science education be saved? *Science,250*, 1327-1330.

Dewey, J. (1938). *Experience and education.* New York, NY: Macmillan Publishing.

Driver, R. (1990). The construction of scientific knowledge in school classrooms. In R. Millar (Ed.), *Doing science: Image of science in science education.* London, England: Palmer Press.

Driver, R., & Bell, B. (1986). Students' thinking and the learning of science: A constructivist view. *School Science Review, 67* (240), 443-456.

Feynman, R. P. (1985). *Surely you're joking, Mr. Feynman.* New York, NY: Bantam Books.

Fisher, D. C. (1991). *Understood Betsy.* New York, NY: Dell Publishing. (Original work published 1917)

Harms, N. C. (1977). *Project Synthesis: An interpretative consolidation of research identifying needs in natural science education.* (A proposal prepared for the National Science Foundation.) Boulder, CO: University of Colorado.

Harms, N. C., & Yager, R. E. (Eds.). (1981). *What research says to the science teacher* (Vol. 3). Washington, DC: National Science Teachers Association.

Helgeson, S. L., Blosser, P. E., & Howe, R. W. (1977). *The status of pre-college science, mathematics, and social science education: 1955-75.* Columbus, OH: Center for Science and Mathematics Education, The Ohio State University.

Hickman, F. M., Patrick, J. J., & Bybee, R. W. (1987). *Science/technology/society: A framework for curriculum reform in secondary school science and social studies.* Boulder, CO: Social Science Education Consortium, Inc.

Hueftle, S. J., Rakow, S. J., & Welch, W. W. (1983). *Images of science: A summary of results from the 1981-82 national assessment in science.* Minneapolis, MN: Minnesota Research and Evaluation Center, The University of Minnesota.

Hurd, P. DeH. (1978). The golden age of biological education 1960-75. In W. V. Mayer, (Ed.), *BSCS biology teacher's handbook* (3rd ed.) (pp. 28-96). New York, NY: John Wiley & Sons, Inc.

Iskandar, S. M. (1991). *An evaluation of the science-technology-society approach to science teaching.* Unpublished doctoral dissertation, The University of Iowa, Iowa City.

Johnson, J. R. (1989). *Technology: Report of the Project 2061 phase I technology panel.* Washington, DC: American Association for the Advancement of Science.

LaRue, R. D. (1988). *Model lessons for secondary social studies classes.* Boulder, CO: Social Science Education Consortium.

Liu, C. (1992). *Evaluating the effectiveness of an inservice teacher education program: The Iowa Chautauqua program.* Unpublished doctoral dissertation, The University of Iowa, Iowa City.

Lu, Y. (1993). *A study of the effectiveness of the science-technology-society approach to science teaching in the elementary school.* Unpublished doctoral dissertation, The University of Iowa, Iowa City.

Mackinnu. (1991). *Comparison of learning outcomes between classes taught with a science-technology-society (STS) approach and a textbook oriented approach.* Unpublished doctoral dissertation, University of Iowa, Iowa City.

Mestre, J. P., & Lochhead, J. (1990). *Academic preparation in science: Teaching for transition from high school to college.* New York, NY: College Entrance Examination Board.

Miller, J. D., Suchner, R. W., & Voelker, A. M. (1980). *Citizenship in an age of science: Changing attitudes among young adults.* New York, NY: Permagon Press.

Montgomery, L. M. (1983). *Emily of New Moon.* New York, NY: Harper & Row. (Original work published 1923)

Montgomery, L. M. (1984). *Anne of Green Gables.* New York, NY: Harper & Row. (Original work published 1908)

National Assessment of Educational Progress. (1978). *The third assessment of science, 1976-77.* Denver, CO: Author.

National Assessment of Educational Progress. (1988). *The science report card: Elements of risk and recovery.* Princeton, NJ: Educational Testing Service.

National Committee on Science Education Standards and Assessment. (1992). *National science education standards: A sampler.* Washington, DC: National Research Council.

National Committee on Science Education Standards and Assessment. (1993a). *National science education standards: A enhanced sampler.* Washington, DC: National Research Council.

National Committee on Science Education Standards and Assessment. (1993b). *National science education standards: July '93 progress report.*

Washington, DC: National Research Council.

National Science Teachers Association. (1990-1991). Science/technology/society: A new effort for providing appropriate science for all (Position Statement). In *NSTA handbook* (pp. 47-48). Washington, DC: Author.

National Science Teachers Association. (1992). *Scope, sequence, and coordination of secondary school science: Vol. I. The content core. A guide for curriculum designers.* Washington, DC: Author.

National Society for the Study of Education, Committee on the Teaching of Science. (1932). *Thirty-first Yearbook: A program for teaching science: Part I.* Chicago, IL: The University of Chicago Press.

Patrick, J. J., & Remy, R. C. (1985). *Connecting science, technology, and society in the education of citizens.* Boulder, CO: Social Science Education Consortium, Inc.

Rutherford, F. J., & Ahlgren, A. (1989). *Science for all Americans.* New York, NY: Oxford University Press.

Spyri, J. (1956). *Heidi.* London, England: Puffin Books, Penguin Group. (Original work 1880)

Voelker, A. M. (1982). The development of an at tentive public for science: Implications for science teaching. In R. E. Yager (Ed.), *What Research Says to the Science Teacher, (Vol. 4)* (pp. 65-79). Washington, DC: National Science Teachers Association.

von Glasersfeld, E. (1987). *The construction of knowledge.* Seaside, CA: The Systems Inquiry Series, Intersystems Publication.

von Glasersfeld, E. (1988). *Cognition, construction of knowledge, and teaching.* Washington, DC: National Science Foundation.

Weiss, I. R. (1978). *Report of the 1977 national survey of science, mathematics, and social studies education: Center for educational research and evaluation.* Washington, DC: U.S. Government Printing Office.

Weiss, I. R. (1987). *Report of the 1985-86 national survey of science and mathematics education.* Research Triangle Park, NC: Center for Educational Research and Evaluation, Research Triangle Institute.

Welch, W. W. (1981). Inquiry in school science. In N. C. Harms and R. E. Yager (Eds.), *What research says to the science teacher* (Vol. 3) (pp. 53-72). Washington, DC: National Science Teachers Association.

Yager, R. E. (1991). The constructivist learning model: Towards real reform in science education. *The Science Teacher, 58*(6), 52-57.

Yager, R. E., Liu, C., & Blunck, S. (1993). *The Iowa Chautauqua program annual assessment report, 1992-93.* Iowa City, IA: The University of Iowa, Science Education Center.

Yager, R. E., Liu, C., & Varrella, G. (1993). *The Iowa scope, sequence, and coordination (SS&C) assessment report, 1990-93.* Iowa City, IA: The University of Iowa, Science Education Center.

Yager, R. E., & Penick, J. E. (1986). Perceptions of four age groups toward science classes, teachers, and the value of science. *Science Education, 70*(4), 355-363.

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Errata

In the October, 1994 issue of *School Science and Mathematics*, the author names for the SSMILes article on page 326, "Get on Track," were reported incorrectly. The names listed as authors were Lloyd H. Barrow, Betty M. Burchett, Paul J. Germann, and Priscilla Callison. The correct author names are Lloyd H. Barrow, Alice Vaughn, Rebecca Litherland, and Laura A. Jackson. Unfortunately, we did not receive these names with the manuscript and misinterpreted the accompanying letter. We deeply regret this incident.

The Examination Where the Student Asks the Questions

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The Examination where the Student Asks the Questions (ESAQ) is an innovative, system-oriented teaching/evaluation strategy ideated, developed and successfully implemented by the author, initially within the teaching of freshman chemistry (general and organic) in a four-year college. It constitutes a reflective, multidimensional response to the urgent need for effective teaching and evaluation strategies which will foster question asking, and higher order cognitive skills (HOCS). Such strategies are, ideally, relevant, challenging, and compatible with the new goals of science teaching and student performance evaluation. The core element of the ESAQ is a prearranged oral examination in which the course professor is examined by students, using home-prepared, written questions. Our experience suggests that the ESAQ can be creatively adopted and successfully implemented in different contexts of science teaching worldwide and would benefit both educators and students. The increase in the HOCS capacity of students is attainable.

It is generally agreed that the development of students' higher order cognitive skills (HOCS), namely, the abilities of critical thinking (CT), decision making (DM), question asking (QA), and problem solving (PS), should become the superordinate goals of science teaching and education (Pizzini, E. L., Shepardson, D. P., and Abel, S. K., 1989; Smith, 1990; Sternberg, 1985; Zoller, 1987a, 1987b, 1990a, 1991).

The rationale of this consensus is that life is, in essence, a continuous process of decision making and problem solving for which students should be prepared through a synthesis of general strategic knowledge and specialized domain knowledge (Zoller, 1987, 1990b). Within this framework, question asking is an essential component of problem-solving ability, that is, the capacity of both asking questions and seeking information *relevant* to a given problem and the skill of applying *critical thinking* in using them within the problem solving process (Zoller, 1987b; 1993).

If, indeed, the QA ability is an integral part of the problem-solving process and an integral part of students' HOCS capacity then, clearly, provisions for students to actually ask questions and exercise this capacity should be provided within the teaching process (Zoller, 1993).

The crucial issue within science education is neither the covering of more advanced domain-specific material nor the increasing of the students' repertoire of useful disciplinary-bound algorithms, but, rather, the translation of the agreed upon objectives into manageable curricula and effective teaching strategies. The examination where the student asks the questions (ESAQ) is one such strategy.

The Examination Where the Student Asks the Questions

The Examination where the Student Asks the Questions (ESAQ) is an innovative, system-oriented teaching/evaluation strategy ideated, developed and successfully implemented by the author, initially within the teaching of freshman chemistry (general and organic) in a four-year college. It constitutes a reflective, multidimensional response to the urgent need for effective teaching and evaluation strategies which will foster question asking and higher order cognitive skills. Such strategies are, ideally, relevant, challenging, and compatible with the new goals of science teaching and student performance evaluation (Bunce et al., 1991; Zoller, 1987a; 1990b, 1991).

The core element of the ESAQ is a prearranged oral examination class session in which the *course professor is examined by class students*. In this contradiction to the traditional "pencil and paper" class examination (in which the students respond to a series of questions/problems prepared and/or assembled by the professor), the students examine their professor orally, using their home-prepared, written questions related to the course. Each student is required to formulate two to three relevant and meaningful questions, one of which is to be used for the class examination. Afterward, toward the end of the examination session, all questions are submitted to the professor for grading. Two to five of the

students' formulated questions (which have not been treated during the class session) are selected by the professor and redistributed to all course participants to serve as a student-designed "take home" examination. Students respond individually to their pool-selected questions at home, and return their prepared answers to their professor for evaluation. Obviously, this is only one of various possible alternative procedures for conducting the ESAQ. For example, the students' answers may be redistributed for peer correction/evaluation and even grading, a process which must be closely guided by the course instructor. Other variations are possible.

Based on our experience, about twenty questions can be adequately "handled" during one-and-a-half ordinary class sessions, including accompanying short discussions and active student participation. If HOCS-oriented teaching styles are used in the course (e.g., problem solving, open-ended questions and higher order cognitive skills-oriented home assignments), many of the student-prepared questions are of the higher level of thinking type. Two examples of students' questions follow:

1. Compound 1 — C_6H_{12} underwent ozonolysis followed by treatment with Zn/HCl in aqueous solution. The two compounds obtained were CH_3CHO and $CH_3CH_2COCH_3$. What is the structure of 1? Rationalize and explain.

2. The di-aldehyde OHC-CH=CH-CHO was treated with Br_2 to obtain the dibromide, the addition product of the bromine to the double bond. Will the product(s) obtained be optically active? Rationalize and explain your answer.

Discussion and Results

The Examination where the Student Asks the Questions clearly requires decision making, question asking and problem (not exercise!) solving skills on the part of the students and, thus, contributes toward the development of their HOCS skills in the context of both the specific content, and processes of the science taught in other domains. It definitely poses an intellectual challenge to students and provides opportunity for dealing with their—and not the professor's—course-related questions and/or misconceptions. As such, it increases the course relevancy to students' needs which, in turn, results in their increased interest and active involvement. Additional benefits of the ESAQ are the almost total elimination of students' test anxiety, the creation of provisions for more involvement of female students in the science learning process, the improvement of the reciprocal student-professor feedback mechanisms, the encouragement of independent, evaluative thinking on the part of students and last, but not least, the positive response to the students' preferred type of examinations (Zoller & Ben-Chaim, 1989).

Although the ESAQ constitutes a challenge for both students and professors, the self confidence of the latter is clearly much more crucial than that of the former for the successful implementation of this innovation.

The Examination where the Student Asks the Questions was implemented instead of traditional midterm examinations in freshman General Chemistry and Introduction to Modern Organic Chemistry courses for biology majors (most of them prospective high school biology teachers), as a complementary component within a comprehensive higher order cognitive skills-oriented teaching style. The innovative idea was accepted enthusiastically by most of the students, albeit not necessarily for the same reasons: some looked forward to the opportunity of challenging the instructor, whereas others felt relieved at not having to take the "difficult" (traditional) exam. However, most students have had the gut feeling right from the outset that this new mode of examination and their necessary preparation before, and related activities after, will result in thorough learning and substantial gains in both their understanding of the course content and HOCS capability.

Interestingly, the mean scores of the students (N=35) for their midterm ESAQ *questions, answers* (to the selected redistributed questions to which they responded at home) and *final* (traditional, oral) exam in the organic chemistry course were *77.2, 79.7* and *71.1* respectively. The interpretation of these results is beyond the scope of this paper. Significant, however, are the similar scores of the students on their questions and answers.

Conclusion

Our experience suggests that the Examination where the Student Asks the Questions (ESAQ) can be creatively adapted and successfully implemented in different contexts of science teaching worldwide and would benefit both educators and students. The increasing of the higher order cognitive skills (HOCS) capacity of students — science and nonscience majors — is the task ahead for science educators. Such an increase is attainable via an appropriate use of ESAQ. Though it is not an easy

task, it *should* and *can* be done. The ESAQ is consistent with current trends and accepted principles of science teaching and its potential for the development of students' HOCS is apparent. Therefore, its implementation and use for the improvement of science teaching should be encouraged. Further studies of the effectiveness of ESAQ in increasing the HOCS capacity of students and its potential in improving the teaching skills of prospective science teachers are in progress.

References

Bunce, D.M., Gabel, D.L. and Samuel, J.V. (1991). Enhancing chemistry problem-solving achievement using problem categorization. *Journal of Research in Science Teaching, 28,* (6), 505-521.

Pizzini, E.L., Shepardson, D.P., and Abel, S.K. (1989). A rational for and the development of a problem solving model of instruction in science education. *Science Education, 73,* (5), 523-534.

Smith, M.U. (Ed.), (1990). *Toward a unified theory of problem solving.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Sternberg, R.J. (1985). Critical thinking: Its nature, measurement and improvement. In F.R. Link (Ed.), *Essays on the intellect.* Alexandria, VA: Association for Supervision and Curriculum Development.

Zoller, U. (1987a). Problem-solving and decision-making in science-technology-environment-society (STES) education. In K. Riqarts (Ed.), *Science and technology education and the quality of life, Vol. 2* (pp. 562-569). Kiel: IPN materialen.

Zoller, U. (1987b). The fostering of question-asking capability - A meaningful aspect of problem-solving in chemistry. *Journal of Chemical Education, 64,* (6), 510-512.

Zoller, U. (1990a). Environmental education at the university: The "problem solving-decision making act" within a critical system-thinking framework. *Higher Education in Europe, 15,* (4), 5-14.

Zoller, U. (1990b). The IEE—an STS approach. *Journal of College Science Teaching, 19,* (5), 289-291.

Zoller, U. (1991). Teaching/learning styles, performance, and students' teaching evaluation in S/T/E/S-focused science teacher education. A quasiquantitative probe of a case study. *Journal of Research in Science Teaching, 28,* (7), 593-607.

Zoller, U. (1993). Are lecture and learning compatible? Maybe for LOCS; Unlikely for HOCS. *Journal Chemical Education, 70* (3), 195-197.

Zoller, U. and Ben-Chaim, D. (1989). Interaction between examination type, anxiety state, and academic achievement in college science: An action-oriented research. *Journal of Research in Science Teaching, 26,* (1), 65-77.

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Call For Papers

The 1997 Yearbook of the National Council of Teachers of Mathematics will focus on equity. *Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity* will provide a vision of how research and practice can ensure a powerful mathematics program for all students regardless of their gender, race, ethnicity, or socioeconomic situation. Interested individuals can obtain a copy of guidelines for the preparation of manuscripts by contacting:

Margaret J. Kenney
Mathematics Institute
Boston College Chestnut Hill, MA 02167-3814
617/552-3775

Due date for receipt of manuscripts is February 15, 1995.

Putting the Fun Into Functions Through the Use of Manipulatives, Computers, and Calculators

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For six years, the authors have conducted a series of professional development workshops for teachers in Grades K-8. This article summarizes some of the research from those workshops and gives an overview of the approach taken. The institutes emphasized basic reasoning skills based upon the NCTM Standards and the Kentucky Education Reform Act, integrating manipulative materials and technology. The article demonstrates the integration of manipulatives and technology using patterns, relations and functions as examples.

"Johnny told me he's never liked math before."

"Ann Marie's mother said she's doing much better at math this year."

"I've never had so much fun teaching math!"

The reason for these comments? Enthusiastic teachers in Grades K-8 were basing their mathematics lessons on the use of manipulatives, supplemented by appropriate calculator and computer activities.

Mathematics Institutes for Teachers

Encouraging teachers to build their lessons on hands-on activities was one of the goals of the Mathematics Institutes for Teachers, conducted at Northern Kentucky University during the summers of 1989 through 1994. Funded by grants from the U. S. Department of Education and the Eisenhower Mathematics and Science Education Act, the institutes strove to promote mathematical reasoning through the integration of manipulatives, calculators, and computers into the mathematics curriculum. The 1989, 1990, 1993, and 1994 institutes were for teachers and administrators in Grades K-4; the 1991 and 1992 institutes focused on Grades 5-8.

Prior to the institutes, teacher-participants completed surveys concerning their use of manipulatives, calculators and computers in their teaching of mathematics. Table 1 shows the percentages of teachers using each of these tools less than once a week. Data from the first three years was typical of the entire program. To improve this situation, participants worked with calculators, a variety of manipulatives, and technology including interactive video ("Project Vision," a program developed for Kentucky's primary grades), turtle robots, LEGO TC Logo, a modem, and adaptive computer devices designed for students with special needs.

The first activities in the institute emphasized basic reasoning skills such as identifying attributes, comparing, sequencing, recognizing patterns and relationships, extending patterns, hypothesizing and hypothesis testing. These reasoning skills were then extended to include number sense, estimation, basic facts and computation, followed by geometry, measurement, graphing, probability and statistics. Activities were designed to demonstrate how manipulative materials and technology could be integrated to help students create their own understanding of ideas. An example of this approach for the primary grades, based on patterns, relationships, and functions, follows.

Table 1. *Percent of Participants Using Manipulatives, Calculators, and Computers Less Than Once a Week: Prior to Institute*

		Manipulatives	Calculators	Computers
Group 1:	Elementary (1989) (n = 103)	37	88	49
Group 2:	Elementary (1990) (n = 89)	47	97	59
Group 3:	Middle School (1991) (n = 40)	84	63	74

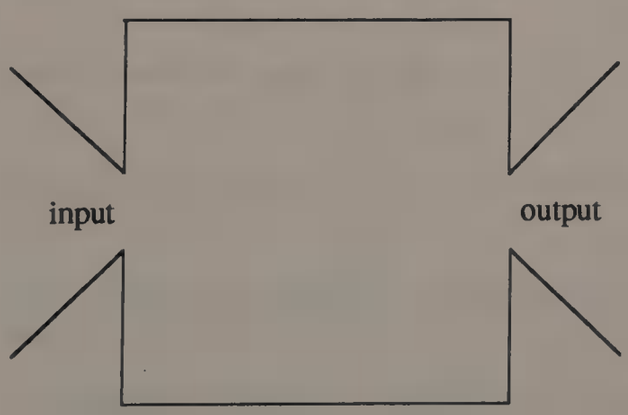
Patterns, Relationships, Functions

The *Curriculum and Evaluation Standards* (1989) of the National Council of Teachers of Mathematics calls for the study of patterns and relationships in Grades K-4 so that students can "... represent and describe mathematical relationships" and "explore the use of variables and open sentences to express relationships" (Standard 13, Grades K-4, p. 60). Such explorations involve "Concepts of Whole Number Operations" (Standard 7) and lead easily to the study of functions, explicitly recommended in Standard 8 for Grades 5-8. The *Standards* recommend that students in these grades should be able to "analyze functional relationships to explain how a change in one quantity results in a change in another" (Standard 8, Grades 5-8, p. 98). In addition, the *Kentucky Curriculum Framework* includes the academic expectation of understanding the concept of "change" in mathematics for all students in Grades K-12. Functions provide students with a good way to look at the concept of change, and this concept was included for the participants in the K-4 Institute as well as those in the Middle Grades Institute.

Use of Manipulatives

The topic of numerical relationships and functions was introduced through the use of a handmade "function" machine. This machine consists simply of a drawing, such as the following, and some materials to be used with the machine. The teacher should have one copy of the drawing for the overhead projector and each student should have one on his or her desk.

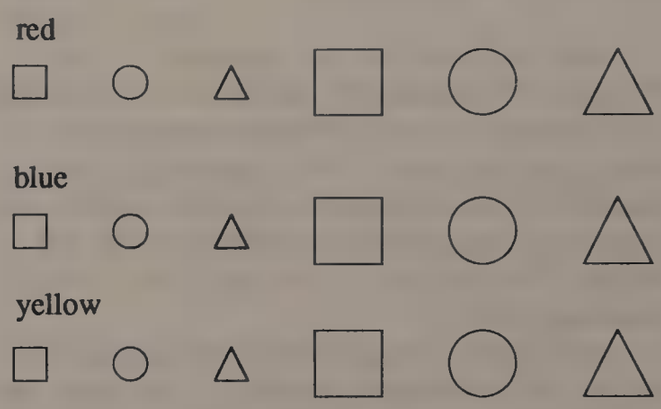
Figure 1. *Function machine for teacher and students.*



Taught abstractly, functions can be very confusing to young children, but these same children can understand the concept of a function quite well when they begin by using concrete materials and then move to the use of calculators and computers.

Young children may begin with nonnumerical rules using attribute materials. Two students should be selected to demonstrate a rule on the function machine with an overhead projector or on a magnetic blackboard with magnets attached to the attribute pieces. Begin by using a set of attribute materials that differ in color, shape and size. You might have 18 pieces that are red, blue or yellow; large or small; and squares, circles or triangles. (See Figure 2)

Figure 2. *Attribute materials that differ in color, shape and size.*



The first student makes up a rule which the child keeps to him/herself such as, "change color only." The second student then selects a piece such as the large red triangle and places it in the input side of the machine. The rule-making student then selects either a large blue triangle or a large yellow triangle for the output. Another block is selected and placed in the input, and the class attempts to predict the output. Play continues until the class is certain of the rule. When students have become proficient guessing a simple rule, they should try more complicated rules such as "change size and shape." After playing the game as a whole class, children can break into smaller groups to play on their own, then get back together as a whole class to discuss any strategies discovered as they played.

After students have used a function machine with attribute materials, they may use the machine in a similar fashion with chips. The following represents one scenario by which students might be introduced to numerical functions.

Danny: Selects 4 chips for the input in the function machine and places these chips in the input on the overhead projector, shown to everyone in the class.

Cara: Secretly decides upon a function such as "add 5," performs the function on the chips and places the new amount of chips (in this case 9) into the output for everyone to see.
Will: Comes up to place a new input into the machine.

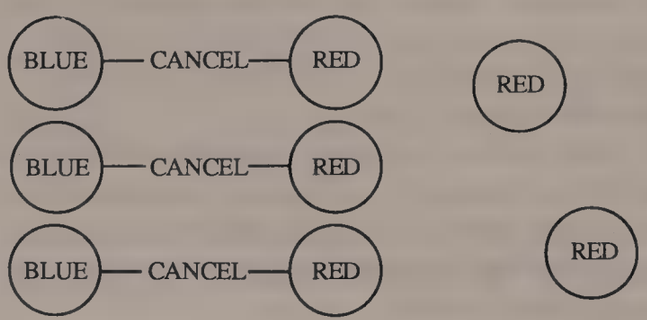
Whole Class: Guesses what the new output will be.

Cara: Performs the same function on Will's chips and places the output in the output section of the machine.

This should continue until everyone in class thinks he or she knows what the function is. Students then check their responses with the student who made up the function. After the students are comfortable with the function machine, introduce the use of charts to keep track of the children's inputs and outputs. The chart will be necessary as the rules get more complicated.

Students should begin with simple functions, such as adding a single digit number to the input. Be careful with subtraction for the function—it will frequently yield a negative number for the output. However, this may not be a problem for children as young as first grade who have been introduced to negative numbers. On the function machine, students can use blue chips to represent positive numbers and red chips to represent negative numbers. Students learn that 1 red chip cancels 1 blue chip. If a student has 3 blue chips for the input, and the function is "add a negative 5," the students learn to add 5 red chips, cancel 3 of them with the 3 blue chips and have 2 red chips for the answer. (See Figure 3)

Figure 3. *Chip illustration of 3 + -5.*



A similar problem arises when division is used with older children and the answers are not whole numbers. It is difficult to show parts of chips or counters when using plastic ones that cannot be broken. The rule may be established that just the whole number part of the answer needs to be shown.

After children have experienced using a single function, they may start using two functions, such as "add 2 and then multiply the result by 3." This is a good time to discuss whether the order of the operations makes any difference. Students should use the chips to demonstrate what happens if they first add 2 and then multiply by 3. Compare this to the result of multiplying by 3 first, and then adding 2. Encourage students to try out a variety of rules as they play the game in small groups.

Use of Calculators

After students have used the function machine with concrete materials, they can move on to using calculators. Using one calculator for a pair of students, ask the first student to write down a secret function. The second student enters a number into the calculator. The first student then takes the

Figure 4. *Chart for sample function game.*

Round	Input	Output	Comments
1	5	17	Could be: add 12 / mult. by 4, subtr. 3 / mult by 2, add 7
2	2		
3			
4			
5			
6			
7			

calculator, secretly performs the function, and hands the calculator back to the second student with the output showing. The second student then records this on a chart, tries to determine what the function is and then tries a new input, attempting to predict the output before seeing it. Note that the function remains the same for all rounds. One example is shown in Figure 4.

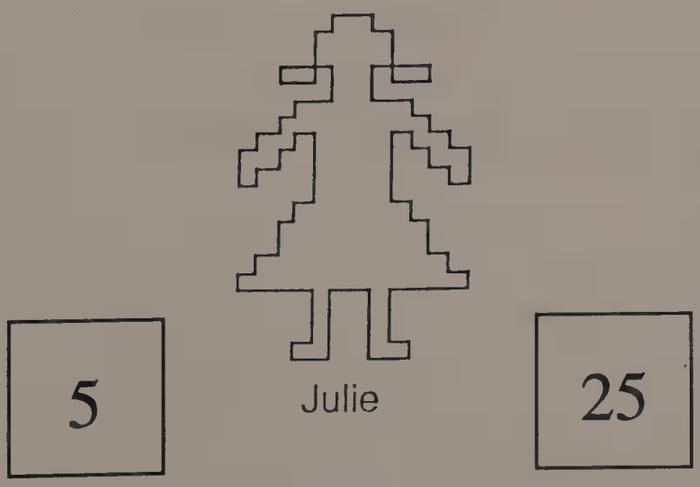
Again, this activity should begin with one operation and progress in difficulty. Younger children should just attempt to determine what the rule is, such as "add 12 to the input;" older students might write the rule in function form such as $f(x) = x + 12$. After the partners have played the game for awhile, get all the students in the class together to discuss strategies and patterns they have discovered while playing.

Computer Programs

A similiar activity can take place using computers and the program "Jane Plus" developed by the Shell Centre, University of Nottingham, England (available from the University of Chicago School Mathematics Program). In this program, the user enters a number and the "function child" on the screen (Jane, Paul, Julie, etc.) changes it according to some undisclosed rule. The task of the user is to discover the rule or function and thus be able to predict the results.

Figure 5. *Example of function problem from computer software* "Jane Plus."

What is Happening?



Write down at least six different things that Julie might be doing to turn 5 into 25.

Teachers can adapt the level of difficulty to their classes by choosing the range of numbers and the operations to be used.

There are many methods of using this program in a classroom. The entire class can join in the exercise, with the students' guesses being recorded on the board. Students then hypothesize what the rule might be and check their guesses against the next result. For example, if Julie turns a 5 into a 25, her rule could be "add 20," or it could be "multiply by 5." If Julie is next given a 4, the students can calculate (a sneaky way to practice computation) the results of applying each of the suggested rules. Those who think Julie's rule is "add 20" would anticipate that the 4 will change into a 24; those guessing that Julie's rule is "multiply by 5" hope that Julie's result will be 20.

Figure 6. *Illustration of applying different rules to the given number.*

Rule 1: Add 20

Original Number	Is changed to
5	25
4	24

Rule 2: Multiply by 5

Original Number	Is changed to
5	25
4	20

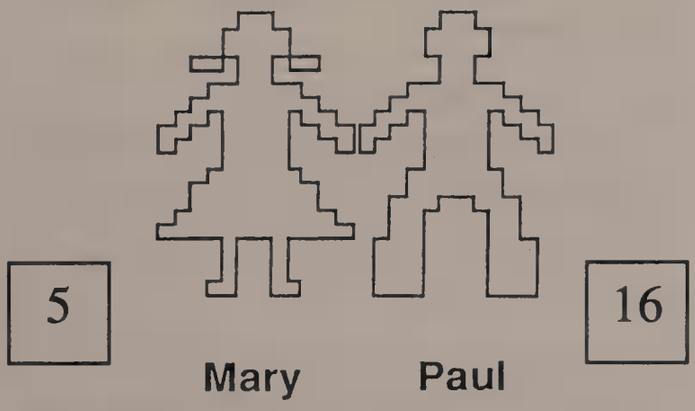
When Julie's result is shown on the screen, students have further information to support or negate their hypotheses. A fruitful discussion could center on whether we can be sure of Julie's rule after two cases or whether we need to examine more cases.

Students could work in small groups to discover the rules. One student could be in charge of choosing the numbers to test, one of suggesting possible rules, one could be assigned to do the computation, one to keep track of results, one to suggest when enough is known to be sure of the rule. Regardless of each student's specific role, all should engage in discussion of the problem.

The program also can be used profitably by individuals. The teacher will need to make sure that students are employing some systematic way of discovering the function and are not just being satisfied with a hit-or-miss approach to obtaining the correct numerical answer.

The computer "function children" also provide an easy approach to a sophisticated mathematical concept. Allowing Mary to operate on a number and then having Paul operate on the result engages the user in determining the result of a "function of a function." For example, if 5 is entered and 16 is the result, it might be that Mary multiplies by 2 and Paul adds 6.

Figure 7. *Example of composite function problem from computer software* "Jane Plus."



Or perhaps Mary multiplies by 3 and Paul adds 1. Students can create several such hypotheses and then check them by using new starting numbers in the program. A further question: Will it make any difference if Paul goes first instead of Mary?

Other software programs also emphasize the concepts of relationships and functions. One of the "Number Munchers" games (published by MECC) requires the user to recognize multiples of a fixed number before being attacked by a "Number Muncher." Recognizing a multiple of 3 requires seeing relationships and patterns and is really the same as identifying x in the function $f(x) = 3x$.

Arithmetic and geometric sequences also provide practice in recognizing and extending patterns. How does a student decide that the next element in 4, 8, 16, 32 . . . is (probably) 64? He or she recognizes a pattern: Each number is twice the preceding one. Or, taking a given element as x, he or she multiplies x by 2 to find the next element. Similarly, in the sequence 1,4,7,10 . . . he or she recognizes that each number is 3 greater than the preceding one.

Post-Institute Results

At the conclusion of the year following the institute, participants again completed surveys concerning their use of manipulatives, calculators and computers in their teaching of mathematics. Table 2 shows the percentages of teachers who at that time reported using these tools less than once a week; the table also repeats the initial findings from the period prior to the institutes.

The results are encouraging in that the percentage using the tools less than once a week is lower in each case. The change in the use of manipulatives by middle grade teachers is especially notable. (It can be noted that the numbers responding, unfortunately, were also lower.)

Table 2. *Percent of Participants Using Manipulatives, Calculators, and Computers Less Than Once a Week: Post-Institute Compared to Prior*

		Manipulatives	Calculators	Computers
Group 1:	Elementary (1989)			
	Prior to Institute (n =103)	37	88	49
	Post-Institute (n = 54)	17	70	40
Group 2:	Elementary (1990)			
	Prior to Institute (n =89)	47	97	59
	Post-Institute (n = 46)	30	72	39
Group 3:	Middle School (1991)			
	Prior to Institute (n =40)	84	63	74
	Post-Institute (n = 25)	32	24	60

From Institute to Classroom

Since the conclusion of the Summer Institutes, participants have been implementing activities in their classrooms and conducting professional development workshops for other teachers in their school districts. Institute participants who were hesitant to play the role of "expert" with their peers have found that sharing ideas with colleagues is appreciated. Some unexpected discoveries also have taken place. Although over half of the teachers had cited "lack of availability" or "lack of funds" as a reason for not making more use of manipulatives, computers, and calculators, several have reported finding such materials in various storage places at their schools. With a little encouragement, these materials have "come out of the closet" and found their rightful place in the classroom!

References

Boysen, T. C. (1992). *Kentucky's preliminary curriculum framework*. Frankfort, KY: Kentucky Department of Education.

Fraser, R. and Wells, C. (1990). *Patterns and functions*. [Computer program]. Chicago, IL: University of Chicago.

MECC. (1985). *Number munchers*. [Computer program]. St. Paul, MN: Minnesota Educational Computing Consortium.

National Council of Teachers of Mathematics, Commission on Standards for School Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Widmer, C. C. (1989). *Patterns workbook*. Unpublished manuscript, Northern Kentucky University, Highland Heights, KY.

Widmer, F. G. (1989). *Pattern potpourri*. [Public domain software for Apple IIe]. Cincinnati, OH: Author.

Note: A public domain disk "Pattern Potpourri" (F. Widmer, 1989) for the Apple IIe was used for the activities involving pattern recognition retention. Copies of this disk and an accompanying *Patterns Workbook* (C. Widmer, 1989), are available from the authors: Drs. Connie Widmer and Linda Sheffield, Education Dept., Northern Kentucky University, Highland Heights, KY 41099.

How Language Does Confuse!

Bruno, what's a right circular cylinder?
It's a cylinder that's fairly round
That's a right interesting answer, Bruno--- But it's not right!
JN BOYD

If it's not right, must it be wrong?

Empowerment of Elementary School Teachers to Implement Science Curriculum Reforms

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The authors evaluate the Science-Technology-Society (STS) views, attitudes and literacy of Israeli elementary science school teachers in three in-depth, longitudinal inservice programs. The programs were aimed at promoting teacher empowerment for the implementation, on a national scale, of a new STS-oriented elementary science curriculum reform. By using a blend of qualitative and quantitative research methods, the effectiveness of these similar but not identical programs has been determined. It was found that appropriately designed inservice science teacher training courses did induce a shift in the STS views/positions, beliefs/attitudes and literacy of elementary science teachers, and that this shift can be increased by using research-based data, obtained within an ongoing formative education process.

The theme of Science-Technology-Society (STS) has not only made its stride into contemporary science education of late, but has also been recognized by many as an essential component of present and future science and general education for all (Aikenhead, 1985; Bybee, 1987; Yager, 1986; Zoller, 1987; 1992). The guiding principle of the STS movement is that all citizens should be capable of understanding, analysis and evaluation of issues in the modern science/technology/society realms in order to take a meaningful, active part in the democratic decision-making process (Bybee, 1985; Fleming, 1989; Zoller, 1987; 1991).

The need for new directions in science education in response to new needs is uncontested. A major issue of concern, however, is the problematic nature of the reform process designed to facilitate the institutionalization of advocated STS curriculum reforms (Miles, 1983; Miles, Ekholm & Vandenberghe, 1987; Thompson & Deer, 1989).

It was recently argued that in order for the STS-directed change to be successfully institutionalized, a new method of reform should be adopted—one that is critical of the conditions creating theory/practice discontinuities and empowers teachers as researchers and *participants* in the decision-making process (Hart & Robottom, 1990). Moreover, teachers must be directly involved with science educators in reexamination of the most fundamental curricular issues, because they hold underlying assumptions and ideologies which, in turn, are "translated" into actual teaching. Just as students should be involved in STS-related social decision making, so teachers should be involved in the STS-related educational decision-making process. In short, the crucial role played by teachers in effecting any change in school science curriculum, particularly in major curriculum reform initiatives, is well established and highly appreciated (Aikenhead, 1986; Simpson, Hungerford & Volk, 1988; Tobin & Fraser, 1987; Zoller, 1992). In view of the current initiative in reforming science curricula towards the attainment of STS or STES (Science-Technology-Environment-Society) literacy by *all* students (Adams, 1990; Zoller, 1992), a major issue of concern is the development and implementation of effective ways to prepare teachers to use the new pedagogy. Clearly, appropriate pre- and inservice teacher training programs, supported by policies that promote teacher empowerment in the spirit of the intended change, constitute a precondition for success (Ebenezer & Zoller, 1991).

A recent major reflection of the STS movement in Israel is the elementary science curriculum reform initiated by the curriculum branch of the Ministry of Education and Culture. A comprehensive new science curriculum for the primary school has been developed entitled *Science in Technological Society* (abbreviated "MABAT" in Hebrew), aimed at providing scientific literacy to the *entire population* within the framework of the mandatory studies of all primary school pupils (Ministry of Education & Culture, 1988). The three central principles of the program: (a) focusing on science education, (b) extension of science education to contents and concepts associated with applied aspects of science and technology, and (c) dealing with the societal meaning of science and technology. They are reflected in its main objectives, which are in full accord

with those agreed upon by STS proponents worldwide:

- to provide students with knowledge and understanding of phenomena, processes and central principles in the natural sciences and technology;
- to develop within students competencies associated with inquiry, critical and creative thinking, in problem-solving and decision-making processes;
- to develop a rational and value judgment capability and readiness for taking responsibility;
- to develop in students a comprehensive outlook of science-technology-society relationships.

The implementation of this new STS-oriented science curriculum on a national scale, within the Israeli centralized educational system, commenced with the 1989/90 school year. However, in spite of the tremendous effort on the part of the Ministry of Education to induce the curricular change within the existing centralized educational system, much remained to be done in terms of "converting" science teachers into active and effective agents of the intended change. Clearly, the reorientation of elementary science teachers through adequate inservice teacher training programs is a key to the success of the science curriculum reform. Accordingly, inservice programs for elementary science teachers have been developed and implemented in the last three years. Their objectives, scope, emphasis, and expectations changed constantly through the implementation process of MABAT in the school system.

Objectives and Purposes

This study concerns prototype longitudinal inservice elementary science teacher training programs (IESTTP) conducted in Israel during 1986-1990, aimed at supporting teachers, mainly "biologically-oriented," in reorienting themselves in accordance with the new trends in science teaching. These teachers have, once a week, a day free from teaching commitments during which they are expected to actively participate in inservice relevant teacher training courses organized regionally by the Ministry of Education.

Since teachers' STS views, attitudes and literacy (Ben-Chaim & Zoller, 1991) are "translated" into classroom teaching and practice, we have focused on these aspects. Specifically, we have assessed the effectiveness of the investigated programs in affecting the STS outlook profiles of the participating teachers (Zoller, Donn, Wild & Beckett, 1991). Thus, our study represents a research-based attempt of practitioners to assess the effectiveness of an inservice teacher training program, aimed at promoting teacher empowerment for the implementation of an STS-oriented science curriculum reform. Specifically, we attempted to:

1. Provide a deeper insight into the curricular issues involved with an STS-oriented science curriculum reform;
2. Establish the STS views, attitudes and literacy of elementary science teachers;
3. Determine the effectiveness of specially designed inservice teacher training programs in affecting participants' STS outlooks.

The results and conclusions arrived at may serve both educational policy makers and developers of IESTTPs in designing, planning and implementing such programs.

Comparative Description of the Three Inservice Training Programs (IESTTPs)

The first IESTTP, designated by us as "Nature Teacher Leaders' Course" (1986-1989) was a three-year (720 hours) course, 87% of which was devoted to the study of the traditional scientific disciplines (biology, physics and chemistry, in this order of relative weight) and the remainder to the methods of their teaching. The emphasis was on scientific knowledge and inquiry skills, based on the existing prereform science curriculum in primary schools. The objectives of that curriculum were " . . . to provide children with knowledge and understanding of principles, processes and important phenomena . . . and to develop their skills associated with the inquiry processes and scientific work." The relevance of the school science to daily life and problems was not considered as important and, consequently, was not reflected nor emphasized either in the curriculum or in the teacher training program. Similarly, relatedness to the environment was not particularly emphasized, in either the curriculum nor in the program, and was left to the individual teacher's consideration.

The second IESTTP designated by us as "Nature/STS Course" (1989-1990), was a one-year (224 hours) STS-oriented course, which was devoted to the study of science (physics, earth sciences and some biology), field studies (science & technology), computers, and STS-related didactic guidance (methods), in 56%, 19%, 12.5% and 12.5% of the

course hours, respectively. The relative weights of teaching science in the two courses reflected the fact that the latter was aimed at preparing teachers toward absorption and teaching the new curriculum, whereas the main purpose of the former was to provide the teachers with basic scientific knowledge. Significantly, about one third of the time of the second course (31%) was devoted to STS issues, compared to about 13% in the previous course. Furthermore, there was more emphasis, within the "didactic guidance" section of the second course, on STS-related topics and teaching strategies, than there was in the "methods" section of the first one.

The third IESTTP (started in 1990) designated by us as "STS Course," was also a one-year (224 hours) course, the time division being very similar to that of the second course in the previous year.

However, in order for the program to further enhance the participants' empowerment to become active change agents of the new STS curriculum (MABAT), the following elements were either strengthed and/or added: (a) more technological applications in the energy work domain; (b) special guest lectures and discussions dealing with the philosophies of science and technology, as well as assessment; (c) actual working on the new MABAT curriculum as an integral part of both the course and the participants' teaching duties in their elementary schools; (d) field trips to industrial complexes, applied research institutes, an electrical power station, and environmental monitoring station; (e) "hands-on" experience of integrating the computer in the actual teaching of the new curriculum; (f) special sessions aimed at developing the participants' skills to guide their peers to implement the new STS-oriented science curriculum. All the above were expected to increase participants' knowledge, confidence and capacity in dealing with STS issues as vital components of their empowerment.

Methodology and Procedures

A blend of qualitative and quantitative statistical approaches was employed to follow the pattern of the subject teachers' arguments and rationale concerning their STS views and attitudes in relation to the new primary science curriculum. This is in line with current trends in science education research and the agreed-upon importance and complementarity of the qualitative and quantitative research paradigms in dealing with science educational concerns (Roberts, 1982). The qualitative part is based mainly on the application of the "depth interview" method (Jones, 1985) in the form of a directed, open class discussion on the STS issues included in a questionnaire to which the participants were previously asked to respond. The quantitative method was used to (a) establish the STS outlook profiles of the participants in the inservice courses, and (b) test the existence (or nonexistence) of differences in the STS views, attitudes, and literacy among participants in the three teacher training courses mentioned above.

The research population consisted of 78 elementary science teachers. There were 28 (21F; 7M), 24 (20F; 4M), and 26 (24F; 2M) in the Nature Teacher Leaders (NTL), Nature/STS (NS), and STS courses, respectively. Most of the participants in the last two courses were graduates of a previous intensive inservice course of nature studies for senior teachers, having various lengths of teaching experience at all types of urban and rural elementary schools in the northern part of the country. Most of the participants of the first course, however, received their "seniority" as a result of their taking the NTL course.

A condensed questionnaire, comprised of six representative statements selected from the VOSTS Inventory Form CDN.mc.4 (Aikenhead, 1987), constituted the research instrument for assessing course participants' STS views/positions (Questions 1 & 2 Category 1), beliefs/attitudes (Questions 3 & 6 Category 2), and literacy (Questions 4 & 5 Category 3). This short questionnaire had been successfully applied previously to assess the STS-related views of both students and teachers (Ben-Chaim & Zoller, 1991; Zoller, Donn, Wild, & Beckett, 1991).

Following are the six items/statements used in the questionnaire. For the sake of brevity, the optional positions which accompanied each item/question are provided here for Questions 1, 2, 3 and 6 only. The last three responses of each item are the same for all items, that is: I don't understand; I don't know enough about this subject to make a choice; and none of these choices fits my basic viewpoint.

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1. Scientists and engineers should be the ones to decide on world food production and food distribution (e.g., what crops to plant, where best to plant them, how to transport food efficiently, how to get food to those who need it, etc.) because scientists and engineers are the people who know the facts best.

Your position, basically (Please choose one.).
Scientists and engineers should decide because:

A. They have the training and facts which give them a better understanding of the issues.

B. They have the knowledge and can make better decisions than government bureaucrats or private companies, both of whom have vested interests.
C. They have the training and facts which give them a better understanding but, the public should be informed and consulted.
D. The decision should be made equally; viewpoints of scientists and engineers, other specialists, and the informed public should all be considered in decisions which affect our society.
E. The government should decide because the issue is basically a political one but, scientists and engineers should give advice.
F. The public should decide because the decision affects everyone but, scientists and engineers should give advice.
G. The public should decide because the public serves as a check on the scientists and engineers. Scientists and engineers have idealistic and narrow views on the issue and thus pay little attention to consequences.

2. Israeli scientists should be held responsible for the harm that might result from their discoveries.

Your position, basically (Please choose one.).

Scientists should be held responsible because:

A. It's part of a scientist's job to ensure that no harm comes from a discovery. Science should cause no harm.
B. If a discovery can be used for both good and bad purposes, the scientists must promote the good use and stop the bad use.
C. They must be aware of the effects of their experiments ahead of time. Science should cause more good than harm.
D. The responsibility should be shared about equally between the scientists and society.

Scientists should not be held responsible because:

E. It's the people who use the discoveries who are responsible. Scientists may be concerned, but they have no control over how others use their discovery.
F. The results of scientific work can't be foreseen (we can't predict if the results will be harmful or not). It's a chance we have to take.
G. Otherwise scientists would quit doing research and science would not progress.
H. Once a discovery is made, others should check its effects. The scientist's job is only to make the discoveries. Science and moral questions are completely separate.

3. Defining what technology is can cause difficulties because technology does many things in Israel.

Your position, basically (Please choose one.).

Mainly technology is:

A. Very similar to science.
B. The application of science.
C. New processes, instruments, tools, machinery, appliances, gadgets, computers, or practical devices for everyday use.
D. Bombs, military hardware, nuclear reactors, etc.
E. Robotics, electronics, computers, communications systems, automation, etc.
F. A technique for doing things.
G. A way of solving practical problems.
H. Inventing, designing, and testing things (e.g. artificial hearts, computers, space vehicles).

4. In order to improve the quality of living in Israel, it would be better to invest money in technological research rather than scientific research.

5. When scientists disagree on an issue (e.g., whether or not low-level radiation is harmful), they disagree mostly because one side does not have all the facts. Such scientific opinion has nothing to do with moral value (right or wrong conduct) or with personal motives (personal recognition, pleasing employers, or pleasing funding agencies).

6. Certain personal characteristics can be important in science (e.g., being open-minded, logical, unbiased, objective, honest). In their research work, scientists display these personal characteristics more than other Israelis do at work.

Your position, basically (Please choose one.).

A. Scientists display these personal characteristics more, because science is more important to the future of society than other professions. If scientists didn't work that way, disasters might occur.
B. Scientists display these personal characteristics more because of the nature of their work. Most other Israelites at work aren't that way as much.
C. Everyone must have these personal characteristics as much as possible in their work, scientists and nonscientists alike.
D. Scientists display these personal characteristics less, and they can also display the opposite characteristics (being narrow-minded, creatively illogical, biased, subjective, and less than honest), because they are so deeply involved, interested, and educated in their specific field.

Seven experts in the field of science and technology education (university professors) were requested to group into "clusters" the several possible responses to each statement. The guiding criterion for the grouping was that each "cluster" should, in principle, express the same view/attitude on the issue dealt with. A cluster was formed based on high agreement between the experts. In those cases of disagreement, a single-response "cluster" was included. Based on the analysis and judgment made by this panel of experts, the six statements of the questionnaire were categorized and grouped into one to four possible responses, for example, ABCD, EFG, etc. (Ben-Chaim & Zoller, 1991; Zoller, Ebenezer, Morley, Paras, Sandberg, West, Walters, & Tan, 1990; Zoller et al., 1991).

A χ^2 test for the difference ($\alpha = 0.05$) between the "STS cluster profiles" of the three courses' participants (i.e., the NTL, NS and STS groups) was conducted for each of the six items. The percentages of each course's participants who selected a specific response as expressing their position on that item were also recorded for some selected responses to each of the six questions. The analysis and interpretation of these data complemented that of the recorded open class discussions of the same STS-related issues, which followed the administration of the "short version" VOSTS questionnaire. The latter was administered toward the end of the NTL and NS courses and right at the beginning of the STS course, thus serving as posttest for the first two and as pretest for the third.

Monitoring followed by interpretive analysis of class discussions—of participants' expectations from the course at each first course meeting and summative evaluation of the course at its end—constituted the third component of this integrated qualitative-quantitative study.

Since STS education is not "neutral" (Zoller, 1987; 1991; 1992), we have determined, based on both the accepted STS philosophy and the courses' rationale, what outcomes in either the quantitative statistical or the qualitative interpretative parts should be considered as "positive/desirable." The data collected and discussions recorded are presented, analyzed and discussed accordingly.

Results and Discussion

Selected representative, characteristic excerpts of the participants' comments in the opening meetings of the NS and STS courses, concerning their expectations from the courses, are given below, in response to the first objective of the study:

Participants of the NS course:

- "I am particularly interested in the integration of the computer . . . and its exploitation in the natural sciences . . ."
- "The integration of the computer in the science curriculum should be emphasized . . . "
- "I am looking for ways of using the computer not as a tool for exercising . . ."
- "I expect from the course an in-depth extension of our knowledge in all subjects of nature, science and technological development . . ."
- "Getting to know curricula which are integrated in MABAT, and methodological ways for their teaching, should be the focus of the course."
- "I would like to learn how to interweave the sciences in the studied subjects in an integrative way . . ."
- "Knowing about contemporary and future applications of advanced technologies is clearly important . . ."

Participants of the STS course:

- "I wonder how is it possible to cover such a comprehensive program (MABAT) in two weekly class sessions . . . I expect to acquire knowledge and didactical means primarily in the areas that I am not familiar with . . . Most important: how to use the computer in teaching? . . ."
- "It is important that we shall have feedback from schools that already teach the program; what was, and what wasn't successful and how they overcame the difficulties . . ."
- "Since there is a mass of material to be taught in too short a time, it is important that in this course we shall get suggestion(s) for a minimum continuum' for selection that would fit everybody . . ."
- "The big classes we have in schools and the lack of nature labs, create a problem which has to be addressed here: how to implement the program within such frameworks . . ."
- "I am not sure that we (the teachers) have the means to transfer this technology to children. The question, the answer to which should be given here, is: How to apply this subject with the aid of the computer?"
- "I expect to get the "MABAT" curriculum booklet which cannot be purchased thus far . . . "
- "Some work on the development of scientific

thinking would be useful, as well as on the interweaving of the various sciences in the topics studied in an integrated fashion . . ."

The participants of the two STS-oriented teacher training courses appear to be quite pragmatic in their approach, in that "knowing how to do it" is their main concern. There are some differences, however. Learning how to integrate the computer, conceived on face value as central in technology and, therefore, "MABAT" teaching, is probably the major expectation of the NS course participants. This should be combined with learning, in the course, how to integrate the different scientific disciplines and advanced technological applications in their future teaching of the MABAT course. The emphasis of the NS group is clearly on the technical know-how and its didactical application within the teaching of MABAT in the future.

In contrast, the emphasis of the STS course participants is on the didactical aspects of the new program in terms of a teacher-proof recipe, to be provided by the course, which will enable them to cover the MABAT course material in their teaching

Table 1. Response Distribution and Analysis of Responses of Elementary Science Teachers in three courses: "Nature Teacher Leaders" (NTL), "Nature/STS" (NS), and "STS" (STS)—on the six item questionnaire.

Question No.	Cluster of Choices	% of responses NTL	NS	STS	value	P
1	ABC	6.3	10.5	8.3		
	D	87.5	73.7	91.7	6.21	n.s.
	EFG	—	5.3	—		
	HIJ and (-)[a]	6.2	10.5	—		
2	ABC	56.3	63.2	62.5		
	D	6.2	5.3	16.7	4.67	n.s.
	EFGH	31.3	31.5	16.7		
	IJK and (-)[a]	6.2	—	4.1		
3	AB	37.5	63.2	33.3		
	CDE	25.0	—	37.5	14.44	0.025
	FGH	25.0	26.3	12.5		
	IJK and (-)[a]	12.5	10.5	16.7		
4	A	—	10.5	—		
	BCDE	93.8	79.0	95.8		
	FG	—	—	—	5.54	n.s.
	H	—	—	—		
	IJK and (-)[a]	6.2	10.5	4.2		
5	ABC	50.0	42.1	54.2		
	DE	18.8	26.3	29.2		
	F	12.5	15.8	4.1	2.69	n.s.
	G	—	—	—		
	HIJ and (-)[a]	18.7	15.8	12.5		
6	AB	6.2	15.8	25.0		
	C	43.8	36.8	66.7	15.79	0.015
	D	18.8	26.3	—		
	EFG and (-)[a]	31.2	21.1	8.3		

(-)[a]—no response to this item/question.

within the existing constraints of their classes. Thus, although the main expectation of both groups from the training courses is to teach them "how to do it," the emphasis in the NS course is on "knowing of and about" as a precondition for STS course implementation. The emphasis in the STS course, however, is on the actual implementation of MABAT and the necessary specific didactics to successfully accomplish that. The shift in the course content and objectives going from the NTL to the NS and STS courses is in line with participants' expectations.

The objective of the first NTL course was to provide the participants with basic scientific literacy, whereas that of the NS and STS course was to prepare the participants to "absorb" the new MABAT program. In an attempt to attain the objectives of the last two courses, more time was allocated to the relevant didactics, use, and applications of computers, as well as ecology/environment-oriented field trips, including visits to technology sites (e.g., a power station and biotechnological factories).

All the above reflects the recent worldwide paradigm shift in science teaching from basic science only to science and technology in the social context, which is further reflected in the results of the statistical part of this study. However, in all of the three courses, the basic approach, of both participant teachers and course organizers, is that the science teachers are *customers* of the new curricula, not partners in the policy making process. Their job is to get to know the program, understand its rationale, and become efficient agents of the expected change.

The distribution of the NTL, NS, and STS course participants' responses to each item of the questionnaire is given in Table 1, in terms of the relative frequencies of "clusters." Each cluster represents a certain position on the issue dealt with.

The χ^2 analysis of the response distribution reveals a significant difference between the three groups of participants only in Category 2 (Questions 3 & 6), that is, in STS beliefs/attitudes. Some differences in the other categories do exist, but they are not significant. A close inspection of the percentage distribution data in Table 1 indicates that, overall, the STS profile of the NTL group is closer to that of the STS group rather than to that of the NS. Similarly, the STS profiles of the NTL and STS groups are closer to one another than those of the NS and STS. This result could be expected to begin with, since having no STS-oriented "treatment" during the NTL course is not expected to change the STS-oriented views of the participants. Thus, their "post" STS views in the NTL course are expected to be essentially the same as the "pre" STS views of the participants in the STS course. These STS profiles represent the *baseline* STS profile of elementary science teachers in Israel.

In considering clusters AB and CDE for Question 3, we note that about two thirds (63.2%) of the participants in the NS course defined technology as "very similar to science" or "the application of science," compared with about one third of the participants in the NTL (at the end) and STS (at the beginning) courses. In contrast, not even one of the participants in the NS course defined technology as "instruments, tools, machinery, . . . computers, bombs, nuclear reactors, . . . etc.," compared with 25% and 37.5% of the participants of the NTL and STS courses, respectively. Although defining technology as "the application of science" is outdated (and erroneous) the overall *positive* outcome of the STS-oriented IESTTP (NS course) is apparent. Selected excerpts of the STS-course participants at the first course meeting, in response to the guiding question "Is there a difference between science and technology?" shed more light on the issue at point and the confusion involved:

- "Technology is a general name to a technological development of all kinds of things . . . "
- "The use of round rods for transferring objects from one place to another by the prehistoric man is *technique;* the chariot is *technology* . . ."
- "Technology is how we serve ourselves with the aid of science . . ."
- "Science does not change—its laws are fixed—whereas technology is something that is developing . . ."
- "Science refers more to the theoretical part and technology deals with the practical part . . ."

Clearly, the difficulty teachers have in distinguishing between science, technology and techniques is reflected in their belief about "what technology is." The fact that more than one quarter (26.3%) of the NS "graduates," compared with 18.8% of NTL—and none of the STS teachers—believed that scientists "can display characteristics of being narrow-minded, biased, subjective, and less than honest, because they are so deeply involved and interested in their specific field" (Q6; D), can be attributed to a *positive* effect of the STS-oriented NS course. The "misconception" that scientists are "completely disinterested human beings . . . " is quite typical for many students

and teachers alike and its prevalence is rather high (Zoller, 1992).

The response percentages with respect to a few selected specific single possible options in each of the six items/statements of the questionnaire are assembled in Table 2.

Table 2. Response Distribution of Responses of the Three Groups (NTL, NS and STS) for Selected Single Possible Options in each of the six Questions.

Question No.	Option	% of responses NTL	NS	STS
1	D	87.5	73.7	91.7
2	B	6.3	31.6	21.8
	C	43.8	21.0	41.7
	H	12.5	26.3	4.2
3	B	37.5	63.2	33.3
	C	18.8	—	29.2
4	D	50.0	47.4	33.3
	E	25.0	15.8	41.7
5	C	31.3	36.8	—
	E	18.8	15.8	25.0
6	C	43.8	36.8	66.7
	D	18.8	26.3	—

According to the Likelihood Ratio χ^2 test, the only significant differences between the NS group (at the end) and the STS group (at the beginning) are Questions 3 and 6: $\chi^2 = 13.884$, $p=0.003$; and $\chi^2 = 11.665$, $p=0.009$, respectively. These findings are in line with what was shown before in relation to the data presented in Table 1.

In relation to the third objective of the study, it is interesting that about ninety percent of the NTL and STS participants (87.5 and 91.7%, respectively) agreed that decisions about world food production and distribution (Q1) should be made equally by scientists and engineers and the *informed* public (response D), compared with only 73.7% of the NS course participants who agreed with this position. The following relevant excerpts of the STS course participants in the guided discussion, which followed the administration of the questionnaire, shed more light on the issues involved:

- "Everybody involved should decide, the scientist as well as the public; however my view is less important than that of the scientists . . ."
- "The question is how much credit one gives the scientists. For instance, the physician recommends and the patient decides. Similarly, concerning power stations, the expert shall point out the risks involved and the public will decide."
- "Scientists inform or do not inform the public according to their interests . . . they are manipulated by politicians and this is the weak point . . ."

The above rationale correlates quite well with the high percentage of NTL and STS course participants who opted for response D. What is unclear, however, is why the "STS-treated" NS course participants opted less for this option. A higher percentage of the NS course participants (compared with those of the other two courses) opted for the ABC responses, and about 5% (compared with nil in the other two) opted for the EFG responses. This pre-post profile shift may be attributed, at least partially, to the STS-oriented type treatment of the NS course teachers. Whether or not this should be considered a positive shift (if real), depends on the personal point of view of the evaluator.

The response distribution of the NS group (compared to those of the other two groups) in relation to options B, C, and H in Q2, can be easily accounted for in terms of the reasons and rationale provided by the NS participants for the choices they made, during the open discussion which followed the administration of the questionnaire:

- "Scientists should take responsibility for their actions since their task is to improve, not to cause damage . . ."
- "The conscientious responsibility is that of the scientist himself."
- "We should aspire that any discovery will be beneficial. To deliberately invent something harmful—no—but not always is it possible to know ahead of time."
- "There is control and there is a need to control the work of scientists."

Qualitatively, it appears that the "STS-treated" NS course participants are aware more than their "STS-untreated" counterparts of the limitations of scientists on the one hand, and of the responsibility of others in society concerning the use (or misuse) of new inventions on the other hand. More research to

clarify the role of the STS treatment in this respect is still needed.

The last combined quantitative analysis/qualitative interpretation approach refers to the response distribution in Q4 as part of the STS literacy category (3). The percentages given in Table 2 for responses D and E add to 75% for either the NTL or the STS groups, and 63.2% for the NS group. Deduction of these numbers from the corresponding percentage numbers of cluster BCDE give 18.8, 20.8, and 15.8% for the choice of B & C by the NTL, STS and NS course participants, respectively. Since the greater the choice of B and C, (especially, "There is really no difference between science and technology") is an indication of the lower STS literacy of the group, the effect of the STS-oriented IESTTP in the desired direction is apparent. It is not surprising that the difficulty in distinction between science and technology is reflected in the STS course participants' oral responses to Item/Question 4 (" . . . it would be better to invest money in technological research rather than scientific research"):

- "I am not sure that what I think should be done is, in fact, not . . . I guess I should do something about this."
- "In spite of the difficulties, we must have positions in every subject and be prepared to express our opinions in order that they would be taken into consideration . . ."
- "Science and technology . . . one is connected to the other . . ."
- "There is a problem in the given statement formulations which are somewhat limiting. I believe that response D is the correct one since it responds to the integration between science and technology. However, the relative weight of this integration is not necessarily equal . . ."

In all our previous, related studies we did not find significant differences between the STS literacy of STS and non-STS students or even between that of students and their teachers (Ben-Chaim & Zoller, 1991; Zoller et al., 1990; 1991). Therefore, the found contribution of the STS-oriented NS course to the STS literacy of the participants is rather encouraging.

It would be appropriate to conclude this section with two brief comments made by two students at the last summative meeting of the NS course:

- "I have learned a lot of new things . . . however, the coping with the unrecognized was rather difficult . . . I am not sure if the acceptance to this new orientation course should not be contingent on attending an appropriate preceding preparatory type course . . ."
- "I did enrich my general repertoire, but more exposure to the new curricular materials as well as meetings with experienced teachers of the new course are definitely needed."

Interestingly, a survey recently conducted (end of 1992) revealed that the majority of the participants in the third IESTPP not only implement the new STS reform (MABAT curriculum), but also serve as mentors in their schools and regions.

Implications for Science Teaching

We are currently in the midst of major curriculum reform initiatives worldwide, a major one being the switch to STS-oriented science teaching. The new STS focus is associated with new goals of science education aimed at critical thinking, problem-solving and decision-making capacity of all students and their ability, as STS-literate citizens, of taking a meaningful, active part in the democratic decision-making process. A major issue of concern in this respect is the students' positions/attitudes, which are to be based on value clarification and acquired knowledge and understanding. This should be done via relevant science education in terms of goals, content, and teaching strategies, not sociopolitical preaching or indoctrination (Zoller et al., 1991, 1992). After all, *change* of the students' and teachers' "illiterate" positions and attitudes concerning STES-related issues is a major goal of STS education. This change is expected to occur via dignified STS education, not indoctrination as a result of students becoming STS literate through an adequate process of science teaching and learning (Zoller et al. 1991a). Clearly, an acquired STS literacy by teachers and a change of their STS-related views and attitudes accordingly, constitute a precondition to a similar change in their students.

The results of our study suggest that (a) the STS beliefs/attitudes can be changed and the views/positions, and literacy of elementary science teachers can be shifted by means of appropriately designed inservice teacher training courses; (b) the effectiveness of these courses in attaining their goals can be increased by using a research-based data associated with their formative evaluation; and (c) there is still a lot to be done with teachers concerning their STS

literacy if we want our students to be STS literate. The empowerment of elementary science teachers to implement an STS-oriented science curriculum and preparing them to use new pedagogy, may be facilitated by appropriately designed and implemented inservice elementary science teacher training programs. However, this is a lengthy process, which means a longitudinal-type research study of the extent of goals' attainment of similar inservice courses will eventually need to be conducted. To what extent such courses can guarantee the induction of the intended change in school science teaching and its outcomes is still an open question. Particularly so, in view of the claim of many, that in contrast to existing practices, the successful implementation of an innovative curricular program is dependent on the full active participation of the teachers involved in the decision-making process associated with the curriculum reform. In view of the unique "mission-oriented" nature of STS education, our results concerning teacher STS training are encouraging: The "STS empowerment" of elementary teachers not only should be done, it *can* be done.

References

Adams, D.L. (1990). Science education for non-majors: The goal is literacy, the method is separate courses. *Bulletin of Science, Technology & Society, 10*(3), 125-129.

Aikenhead, G.S. (1985). Collective social decision-making: Implications for teaching science. *Bulletin of Science, Technology & Society, 5*(2), 117-129.

Aikenhead, G.S. (1986). Preparing undergraduate science teachers in S/T/S: A course in epistemology and sociology of science. In R.K. James, (Ed.), *Science, technology and society: Re sources for science educators* (1985 AETS Yearbook, pp. 59-64). Columbus, Ohio: Association for the Education of Teachers in Science and SMEAC Information Reference Center.

Aikenhead, G.S. (1987). *Views on science-technol ogy-society*. Form CDN.mc 4. Department of Curriculum Studies, University of Saskatchewan, Canada, S7N OWO.

Ben-Chaim, D. & Zoller, U. (1991). The STS outlook profiles of Israeli high school students and their teachers. *International Journal of Science Teaching, 13*(4), 447-458.

Bybee, R.W. (1985). The Sisyphean question in science education: What should scientifically and technologically literate persons know, value and do as citizens? In R.W. Bybee (Ed.), *Science-technology-society, 1985 Yearbook*. Washington, D.C.: National Science Teachers Association.

Bybee, R.W. (1987). Science education and science-technology-society S/T/S theme. *Science Education, 71*(5), 667-683.

Ebenezer, J.V. & Zoller, U. (1993). Grade 10 students' perceptions of and attitudes toward science teaching and school science. *Journal of Research in Science Teaching, 30*(2), 475-486.

Fleming, R. (1989). Literacy for a technological age. *Science Education, 73*(4), 391-404.

Hart, E.P. & Robottom, I.M. (1990). The science-technology-society movement in science education. A critique of the reform process. *Journal of Research of ScienceTeaching, 27*(6), 575-588.

Jones, S. (1985). Depth interviewing. In R. Walker, (Ed.), *Applied qualitative research*. London: Biddles, Ltd.

Miles, M.B. (1983). Unraveling the mysteries of institutionalization. *Educational Leadership, 41*(3), 14-22.

Miles, M.B., Ekholm, M. &Vandenberghe, R. (1987). *Lasting school improvement: Exploring the process of institutionalization*. Paris: Organization for Economic Cooperation and Development.

Ministry of Education & Culture (1988). The Science in Modern Technology ("MABAT") Curriculum (Hebrew). The Curriculum Branch. Tel Aviv: The Israeli Center of Science Education.

Roberts, D.A. (1982). The place of qualitative research in science education. *Journal of Research in Science Teaching, 19*(4), 277-292.

Simpson, P.R., Hungerford, H. & Volk, T.L., (1988). *Environmental education: A process for pre-service teacher training curriculum develop*ment. UNESCO (ED-88/WS/$), Division of Science, Technical and Environmental Education.

Thompson, H. & Deer, C.E. (1989). The institutionalization of a senior secondary curriculum in New South Wales high schools. *Journal of Curriculum Studies, 21*(2), 169-184.

Tobin, K., & Fraser, B.J., (1987). *Exemplary practice in science and mathematics*. Perth, Western Australia: Curtin University of Technology.

Yager, R.E. (1986). Preparing students for a technological world. *Curriculum Review, 24*(3), 21.

Zoller, U. (1987). Problem solving and decision-making in science-technology-environmental-

society (STES) education. In K. Riquarts, (Ed.), *Science and technology education and the quality of life. Vol. 2.* Proceedings of the 4th International Symposium on World Trends in Science and Technology Education, pp. 562-569. Kiel: IPN Materialen.

Zoller, U. (1991). Problem-solving and the 'problem-solving paradox.' In Keiny S. & U. Zoller, (Eds.), *Conceptual issues in environmental education, pp. 71-88.* New York: Peter Lang Publications, University Press.

Zoller, U. (1992). The Technology/Education Interface: STES education for all. *Canadian Journal of Education , 17*(1), 86-91.

Zoller, U., Ebenezer, J., Morley, K., Paras, S., Sandberg, V., West, C., Walters, T. & Tan, S.H. (1990). Goals' attainment in science-technology-society (S/T/S) education and reality. The case of British Columbia. *Science Education, 74*(1), 19-36.

Zoller, U., Donn, S., Wild, R., & Beckett, P. (1991). Students versus their teachers' beliefs and positions on science-technology-society (S/T/S)-oriented issues. *International Journal of Science Education, 13*(1), 25-36.

Zoller, U., Donn, S., Wild, R., & Beckett, P. (1991a). Teachers' beliefs and views on selected science-technology-society topics: A probe into STS literacy versus indoctrination. *Science Education 75*(5), 541-561.

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Discovering Relationships: Logarithmic and Exponential Functions

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A current focus in mathematics education is the use of the computer as a tool in the exploration of mathematics. The purpose of this article is to present an application of the software tool "Derive" in the exploration and visualization of a relationship between logarithmic and exponential functions.

Exponential and logarithmic functions are often important when modeling natural phenomena. Exponential functions are used to model compound interest, growth and decay in populations and radioactive materials, and in Newton's Law of Cooling. Logarithmic functions are often used to create compressed scales since the logarithm of a number with a base greater than one increases much more slowly than the number itself. Logarithmic scales include the decibel scale for sound intensity, the Richter scale for earthquake intensity, and the pH scale for hydrogen ion concentration. Logarithmic functions are used to model relationships such as the velocity of a rocket at burnout (depletion of fuel supply).

The graphs of these functions allow the student to visualize relationships. Important properties of these relationships can then be determined from the graphs, such as: domain and range, maximum and minimum values, and whether the functions are increasing or decreasing on a given interval. But often students have difficulty distinguishing between the basic graphs and determining important properties of these functions.

The following question can be used to reinforce the student's concept of the graphical representations of the exponential and logarithmic functions:

For what values of b do the graphs of the following equations intersect?

$$f(x)=b^x \quad g(x)=\log_b x \tag{1}$$

This single question causes students to explore and discover a variety of interesting mathematical relationships:

1. How do the graphs of the exponential and logarithmic functions behave for base values in the interval $0 < b < 1$ verses $b > 1$?

2. Since the functions are inverses of each other, where must they intersect?

3. Do the functions intersect for base values in the interval $0 < b < 1$? For base values in the interval $b > 1$?

4. What values of b result in the two functions intersecting at exactly one point, at two points, or not at all?

By exploring these questions, students gain a deeper understanding of the graphical representations of the exponential and logarithmic functions, and how they are related. An unexpected bonus is that the answers to the questions involve the number e.

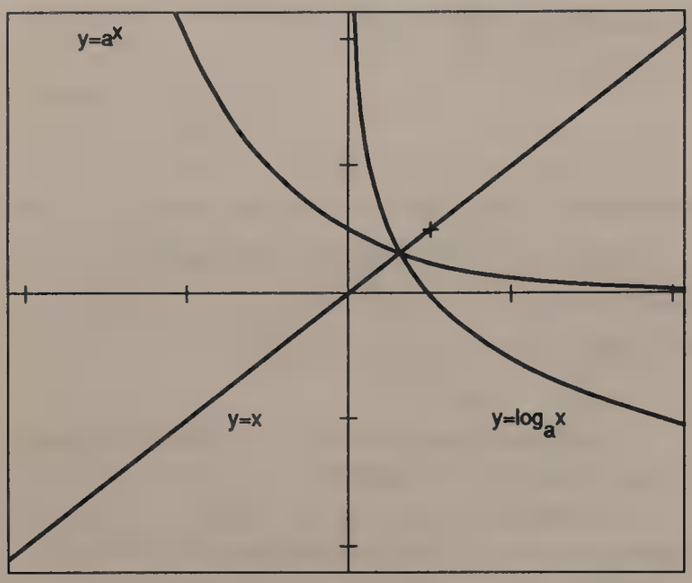
This problem has another interesting property. If the equations are set equal in an attempt to solve for the points of intersection, the following equation results:

$$b^x = \log_b x \tag{2}$$

The equation cannot be solved using basic algebraic techniques! Students see that not all questions can be answered by simple manipulation of variable expressions. The question requires the student to use a different approach to solve the problem. Graphical analysis is one viable alternative.

Graphical analysis requires a degree of accuracy which is best achieved on a computer or on a graphing calculator. Either technology alleviates the burden of point plotting when generating multiple graphs and allows zooming in to approximate values. The student can quickly generate accurate data. Thus, a student's activity is focused on analyzing graphical data and problem solving, not on generating data. The software "Derive" was used to generate the graphs in this paper.

Figure 1. *Graphs of $y=a^x$ and $y=\log_a x$ for a =0.5*



Begin the exploration by investigating the graphs of exponential and logarithmic functions with a base value in the interval $0 < b < 1$. Have students select a base value on this interval. Figure 1 shows the graphs when $b = 0.5$. The line $y = x$ is also plotted. After trying several different base values, students will readily see that for any base value in this interval, the two equations intersect once. That point of intersection will occur on the line $y = x$.

Now have students explore base values in the interval $b > 1$. Figure 2 shows the graphs when $b =$ 1.5 and 2. Many students erroneously conclude that the two graphs never intersect on the interval $b > 1$. Notice in Figure 2 that as the value of b decreases the functions get closer to the line $y = x$. Figures 3 and 4

Figure 2. *Graphs of $y=a^x$ and $y=\log_a x$ for a=1.5 and a=2*

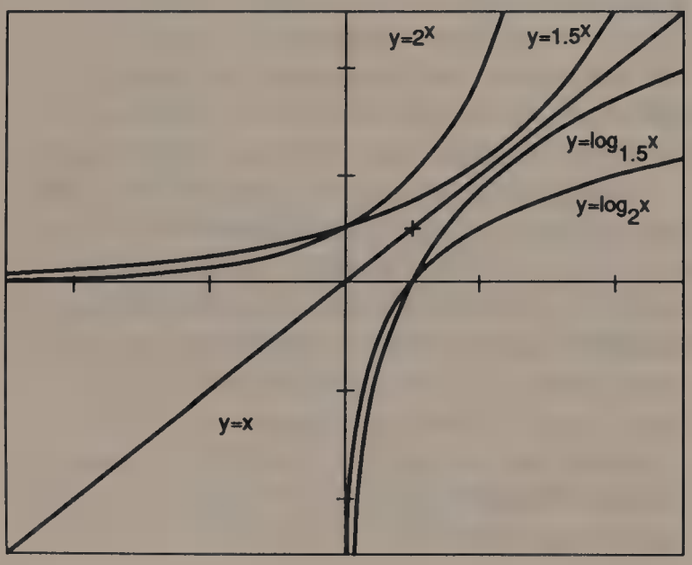
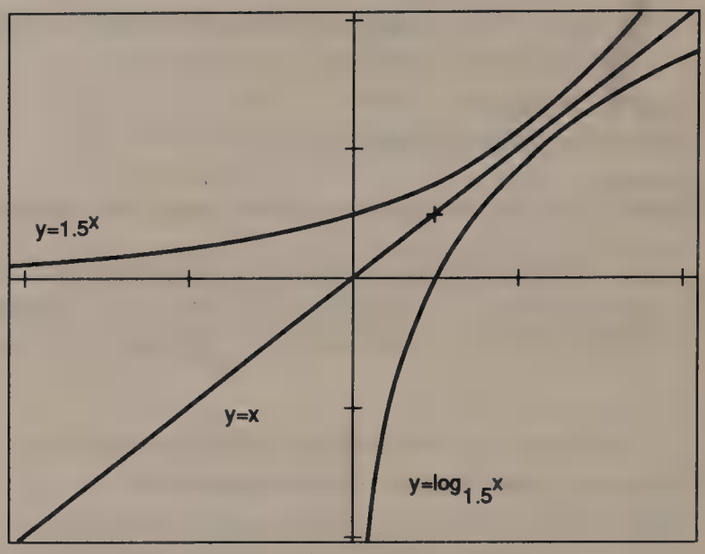


Figure 3. *Graph of $y=a^x$ and $y=\log_a x$ for a=1.5*



demonstrate that the first time the functions intersect on this interval is somewhere between $b = 1.4$ and $b = 1.5$.

To determine the interval of base values where the functions intersect, one needs to find the base value at which exactly one solution exists. The obvious candidate is to let the base value be 1.45, midway between 1.4 and 1.5. Figure 5 shows the graph for a base value of 1.45 at a scale of 2. It appears the functions intersect, but upon zooming in to a scale of 0.2 it is evident that the two functions do not intersect for this base value (See Figure 6).

Now the hunt is on! The new base value is somewhere between 1.4 and 1.45. Continue to select base values and zoom in to see if they intersect or miss. Working together, students can zero in on the approximate base value of 1.4446 as the value where the two functions intersect once (See Figure 7).

Figure 4. *Graphs of $y=a^x$ and $y=\log_a x$ for a=1.4*

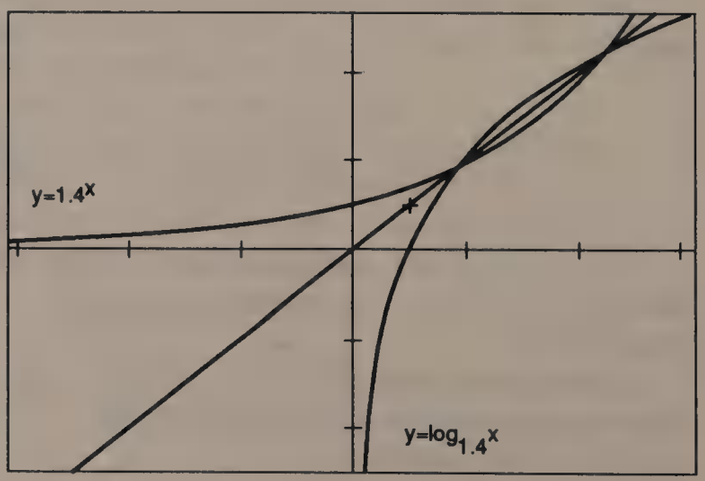
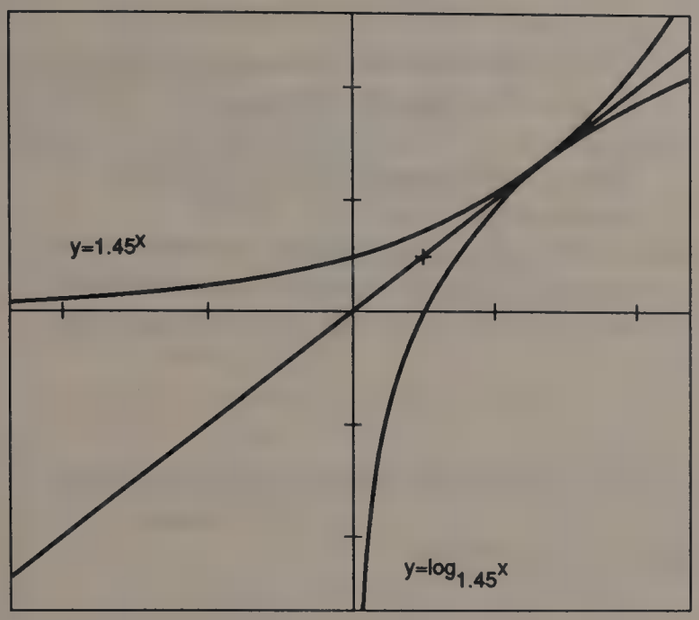


Figure 5. *Graphs of $y=a^x$ and $y=\log_a x$ for $a=1.45$, scale of 2*



With a little insight and knowledge of the number e the exact value can be determined. Use the black cross cursor in "Derive" to zoom in on the point of intersection. "Derive" provides the coordinates of the cross in the lower left hand corner of the graphing screen (see Figure 8). The point of intersection is approximately 2.7194, 2.7187. These values are very close to the approximate value of the number e, 2.71828. One may conjecture that the point of intersection of $y = x$ and the exponential

Figure 6. *Graphs of $y=a^x$ and $y=\log_a x$ for $a=1.45$, scale of 0.2*

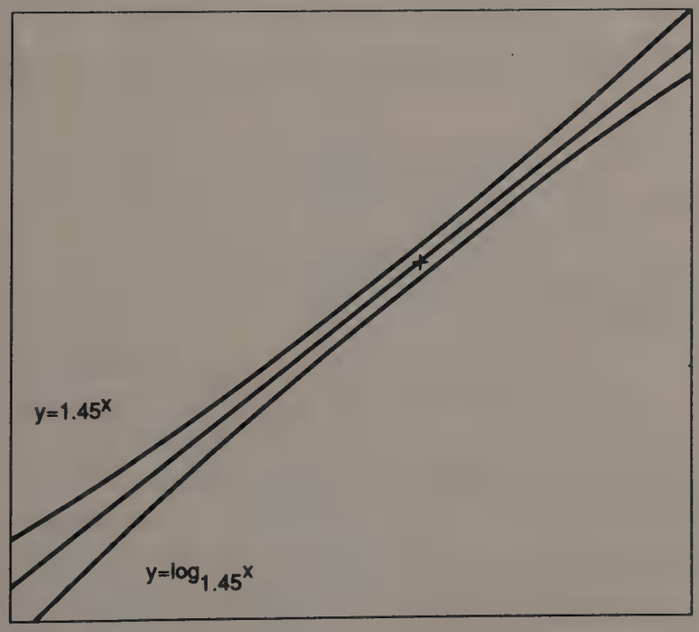
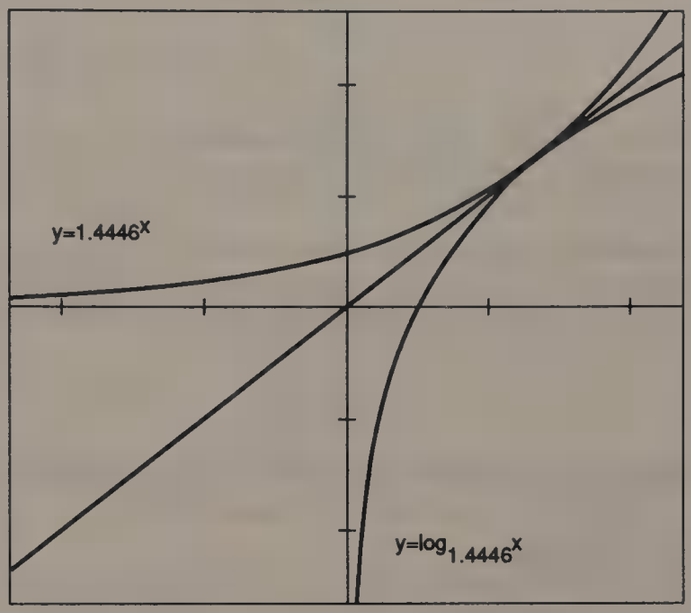


Figure 7. *Graphs of $y=a^x$ and $y=\log_a x$ for $a=1.4446$, scale of 2*



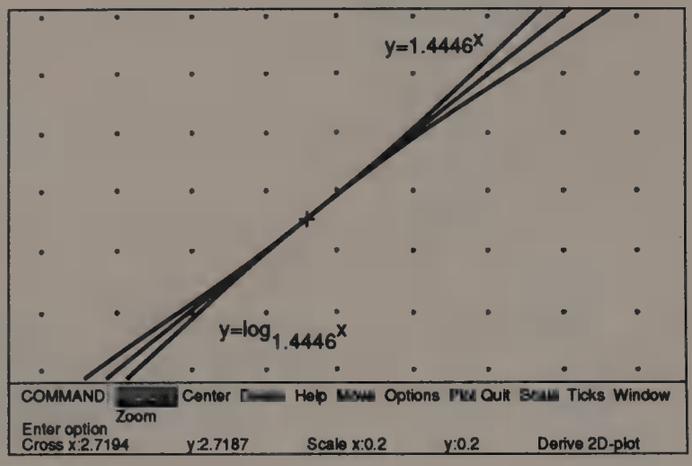
function with the base b is the point (e,e). Substitution into the exponential equation $y=b^x$ gives:

$$e = b^e \text{ thus } b = e^{1/e} \tag{3}$$

This solution can be verified as follows:

$$\begin{aligned} &f(e) = e^{(1/e)^e} \\ &g(e) = \log_{e^{(1/e)}} e \\ &\text{Let } y = \log_{e^{(1/e)}} e \\ &\text{Then } e^{(1/e)^y} = e \\ &y(1/e) = 1 \\ &y = e \\ &\text{Thus } f(e) = e = g(e) \text{ for } b = e^{1/e} \end{aligned} \tag{4}$$

Figure 8. *Graphs of $y=a^x$ and $y=\log_a x$ for $a=1.4446$, scale of 0.2*



This exploration is one example of a class of problems that can be solved by using technology. Software tools such as "Derive" can be used to explore and discover concepts that were previously out of the reach of students, either due to the level of difficulty or the time involved in generating data for the exploration. Many mathematical concepts can be discovered and understood, rather than being provided and memorized. The NCTM *Standards* recommend such an approach. The technology is available to implement it, and the time has come to change our approaches to the instruction of mathematics!

References

Barnett, R., & Ziegler, M. (1989). *College algebra* (4th ed.). New York: McGraw-Hill Book Company.

Mayes, R. (1991). *College algebra sudent laboratories*. Morgantown, WV: West Virginia University.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: NCTM.

Rich, A., Rich, J., & Stoutemyer, D. (1989). *Derive: A mathematical assistant for your personal computer*. Honolulu, HI: Soft Warehouse, Inc.

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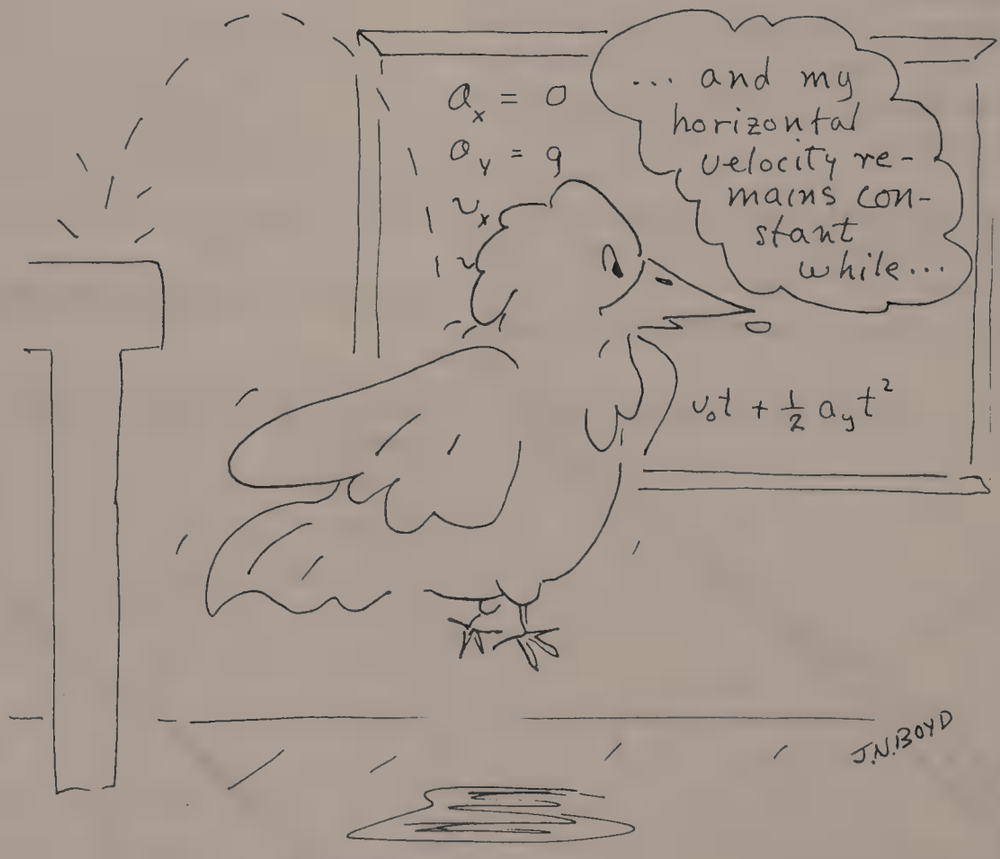
Legends of Avian Mathematics

Plymouth Rock's Lectures Were Always Animated. He Would Often Run Across the Room, or He might Even Leap from a Tabletop to Make his Point. Consequently, his Course Became Known As . . .



Poultry in Motion

Making Mathematical Connections Across the Curriculum: Activities to Help Teachers Begin

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Ample evidence is available to support the contention that, for learning to be meaningful, concepts must be connected and integrated within the experiences of the learner. In mathematics, at least three kinds of connections are particularly beneficial: connections within mathematics, across the curriculum, and with real world contexts. The authors' work with preservice and inservice teachers has convinced them that teachers possess both the willingness and the capability to help students make meaningful connections, given encouragement and support. This article focuses on making mathematical connections across the curriculum; activities which help teachers learn how to design their own are shared.

The notion that, for learning to be meaningful, it must be connected to or "anchored" in something the learner already knows is not new. A succinct statement from reading researchers Pearson and Johnson (1978) describing learning as ". . . building bridges between the new and the known" (p. 24) aptly summarizes the thinking of many researchers in several fields of study regarding the interconnectedness of learning. The need for mathematical connections was articulated as early as 1902 by E. H. Moore in a presidential address to the American Mathematical Society: " . . . everything exists in its relations to other things, and in teaching the one thing the teacher must illuminate these relations" (p. 250). (For a detailed comparison of ideas from Moore's address and parallel recommendations within the *Curriculum and Evaluation Standards for School Mathematics* [NCTM, 1989], see House, 1990.)

According to the *Standards*, a classroom in which making connections is emphasized is characterized as follows:

> Ideas flow naturally from one lesson to another, rather than each lesson being restricted to a narrow objective. Lessons frequently extend over several days so that connections can be explored, discussed, and generalized. Once introduced, a topic is used throughout the mathematics program. Teachers seize opportunities that arise from classroom situations to relate different areas and uses of mathematics. Children are asked to compare and contrast concepts and procedures. They are helped to construct bridges between the concrete and the abstract and between different ways of representing a problem or concept. *Learning and using mathematics are important aspects of the entire school curriculum* [italics added]. (p. 32)

One notable obstacle to realizing such a classroom environment is that teachers have not yet received enough assistance in learning how to plan and implement instruction that helps students make these connections. This kind of teaching is complex, with many facets, and several factors mitigate against its development (cf. House, 1990). Nevertheless, the authors' experience tells them that most teachers are willing and able to develop the necessary strategies, given reasons for doing so and instruction that helps them know how to begin. This article focuses on one of the kinds of mathematical connections recommended in the *Standards*—learning and using mathematics across the curriculum. The authors share several activities they have used as springboards to help preservice and inservice elementary teachers develop their own *Standards*-based curriculum connections. These activities are written to bridge mathematics content for Grades K-4 and 5-8 and are likely to be most appropriate for students at intermediate age levels. The reader is encouraged to adapt them as needed for younger and/or older students. Because the standards for K-4 and 5-8 are loosely parallel, each activity has been identified in relation to a K-4 standard. Since the standards themselves are interconnected, decisions as to which standard each activity addresses should be considered arbitrary.

Activity 1: Trash for Cash

Standard

Mathematics as Problem Solving

Content connection

Environmental Studies

Objective

Students will estimate the amount of trash that could be recycled in their school and/or school system.

Materials

For each group of students: filled trash can from another classroom, sheets of old newspaper, calculator, writing paper, pencils.

Collect filled trash cans from other classrooms. (Keep the sources anonymous.)

Divide the class into groups of four or five students. Distribute sheets of old newspaper and a trash can to each group. Have students spread out the newspaper to protect the floor and dump the contents of the trash can.

Have them sort and classify the contents in as many ways as they can (natural resources used, time needed to replenish these resources, whether there is undue waste involved, length of time needed to decompose, etc.).

Have them reclassify according to whether the products can be recycled, then determine the weight of recyclable materials. Have them predict the number of pounds of recyclable waste disposed of daily in their school and/or school system. (They will need to consider how to sample adequately the trash cans in their school or system, classes that may have more waste than others, number of classrooms and schools, etc.)

Guide the students to investigate recycling opportunities in their community and devise a plan for helping their school or school system participate.

Have students prepare an article for the school newspaper or newsletter that summarizes their findings and enlists the help of others in their recycling efforts. Include any environmental concerns that may have arisen (e.g., too much wasted paper, discarded aluminum cans that could have been recycled, etc.).

A social studies extension for this activity is to have students describe the inhabitants of the classroom from which the artifacts (trash) they examined were collected, using only the information available from the artifacts gathered.

Activity 2: Kentucky (or your state) Logic

Standard

Mathematics as Reasoning

Content connection

Social Studies

Objective

Students will design a logic problem, using knowledge of familiar sites within their state.

Materials

For each group of 4 or 5 students: state map, writing paper, pencils, grid. This activity is appropriate during or after a comprehensive study of your state.

Assign 4 or 5 students to each group. Give them the following tasks:

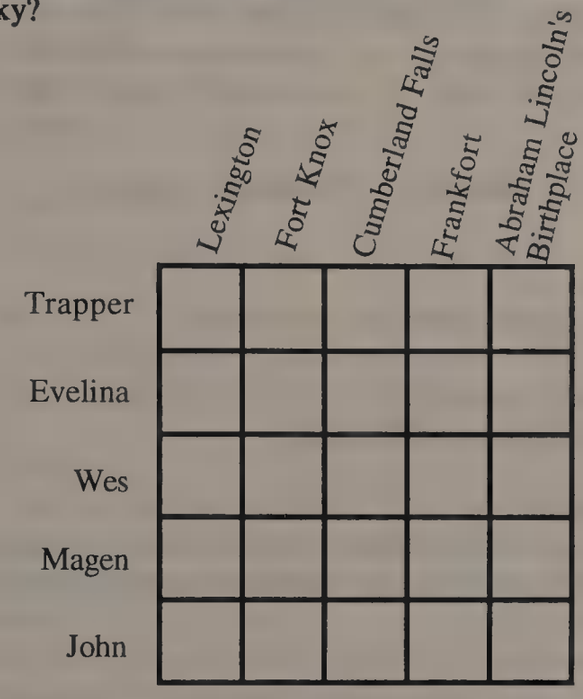
1. Each student will identify a site he or she would like to visit within the state.
2. Each student in the group will list reasons *not* to visit sites identified by other members in the group. These reasons will serve as clues for solving the logic problem they design.
3. A recorder for the group should compile the clues into a list.
4. Students should draw a grid with their names down the side and the sites they have chosen to visit across the top. They should place a check (√) in the block where the grid intersects for their name and place to visit. (They are to choose only one site each.) Have them select from their list of clues as many as are needed to complete their logic problem. Have students eliminate any clues that are unnecessary for solving the problem. (Students may need to work through the problem several times to determine which clues are necessary.)
5. Students should arrange the clues in random order and give to other groups to solve. *Kentucky Logic*[1], designed by a group of fourth grade students at Alvaton Elementary School, Alvaton, KY (Rider,

[1]From "Kentucky Logic" by C. Rider, 1990, *Kentucky Journal for Teachers of Mathematics*, Summer/Fall, p. 26. Adapted by permission.

1990), demonstrates the kind of thinking students are able to do with this activity.

Kentucky Logic

Where would each person like to visit in Kentucky?



Clues

1. Trapper has already met the governor.
2. Wes likes Abraham Lincoln, but he doesn't like the history of presidents.
3. A horse stepped on Evelina's foot one time.
4. John's favorite president is not Abraham Lincoln.
5. Wes doesn't like to watch horses.
6. Trapper and Magen are not very fond of presidential history.
7. Wes is afraid of heights.
8. Trapper doesn't need any more money.
9. Magen and Evelina don't like guns.
10. John has already been to Fort Knox.
11. Trapper hates horses.
12. John doesn't think he should bet on horses.

Activity 3: What's a Dollar?

Standard

Mathematical Connections

Content connection

Mathematics

Objective

Students will represent the value of $1.00 in a variety of ways.

Materials

For each small group of students: coins in plastic bag (equal to $1.00 in value), crayons or markers, grid paper, scissors, magazines or newspapers, writing paper, sheets of newsprint, glue or glue stick, pencils.

Assign students to small groups. Distribute to each group a plastic bag containing coins. (Each bag should contain any variety of coins that total $1.00 in value.) Have students estimate, then count, the amount included in their bag.

Ask the class to list as many ways as they can think of to represent the value of their coins. (Be sure they include lists, tables, paragraphs, bar graphs, pictures of various products that could be purchased with the coins, etc.)

Have students in each group choose three ways to represent the value of $1.00. After they complete their representations, display the work of all groups and compare various ways the value of $1.00 was expressed.

Encourage students to generate additional solutions.

Activity 4: One-Minute Run

Standard

Estimation

Content connection

Physical Education

Objective

Students will compare estimates and actual distances run in one minute.

Materials

For each group of five students: stop watch or watch with second hand; two rocks, bean bags, or small flags (to be used as markers); a set of task cards: Runner, Marker, Timekeeper, Step Counter, Recorder; copy of recording sheet (see below);

One-Minute Run Recording Sheet

	Footsteps	
Name of Student	Estimate	Actual

pencils. For extension: Second copy of recording sheet (for each group); markers in two colors (for the class); long sheet of newsprint (for the class).

Assign students to groups of five and distribute task cards.

Determine a starting point on the playground and ask students to think about how far they can run in one minute.

Have the Runner from each group "step off" the distance of his or her estimated run. The Marker should place a marker of some sort at that point. The Step Counter should count the steps and the Recorder should record the number of steps (see recording sheet).

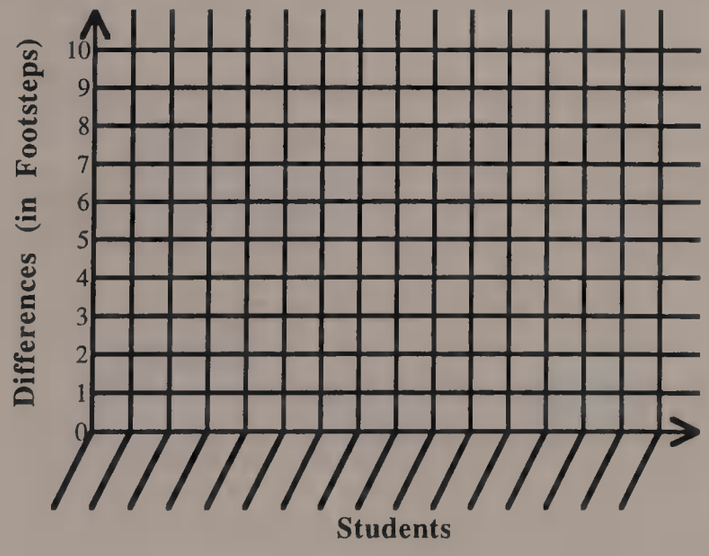
Have the Timekeeper time a one-minute run for the Runner in her or his group. The Marker should place a marker at the Runner's stopping point.

The Runner should then "step off" the actual distance run, the Step Counter should count the steps, and the Recorder should record the number of steps.

Rotate roles until everyone in the group has filled each role.

Have students examine their group's recording sheet and note differences between records of estimated and actual runs. Have them discuss factors that may have affected these differences (e.g., whether the last runners learned to make closer estimates after observing earlier runners, whether some footsteps were longer than others, etc.).

Scatter Plot
Suggested Format



For an extension, repeat the entire activity. Then, using a long sheet of newsprint, have students complete a two-color class scatter plot, recording differences between first-run estimate and actual run in one color and second-run estimate and actual run in another. The students should then look for any patterns in the data and discuss reasons their estimates were more or less accurate the second time.

Activity 5: Newspaper Numbers

Standard

Number Sense and Numeration

Content connection

Informational Reading

Objective

Students will identify uses of numbers in the newspaper and classify them in a variety of ways.

Materials

For each small group of students: newspaper, pair of scissors per person, sheet of art paper, glue or glue stick.

Assign students to small groups and distribute materials.

Have students find all the numbers they can in the newspaper, including those written in standard form, other numerations systems (e.g., Roman numerals), and in words. Have them cut out the numbers, being sure to label what the numbers are used to represent (e.g., 6000 cars, 21 rebounds).

Ask students to think of as many ways as they can to classify the numbers they found (e.g., large, medium-size, and small numbers; odds and evens; numbers representing people and numbers representing things; measurement units). Have them decide on the way they wish to classify their numbers, then glue the numbers on art paper according to the categories they chose.

Provide opportunities for group discussion so that students will recognize that there are many uses of number in the newspaper and a variety of ways to classify the numbers they found. Then display students' work; encourage them to continue to add to their display.

One interesting variation to this activity is to have students omit the names of the categories; classmates should be asked to provide logical names for

the categories by examining the numbers displayed.

As an extension, use Venn diagrams to show overlaps in categories.

Activity 6: "Spatial" Careers

Standard

Geometry and Spatial Sense

Content connection

Career Education

Objective

The students will develop an awareness of the need for spatial sense in everyday life and in careers.

Materials

For each student: construction paper (scraps work well); scissors. For the total group, common objects such as the following: coins (several denominations); Lifesavers; straws (focus on length); milk cartons (focus on the base); playing cards; other readily available objects.

List on the board several common objects that are available but out of view of the students. Encourage students to form a mental image of each, giving attention to both the size and the shape of the object.

Ask students to work individually, representing the mental image formed by cutting the shape of the object from construction paper. Then assign students to pairs.

Have them select from their two representations the one that they both agree is closest to the size and shape of the actual object. Distribute the actual objects and have students compare each representation with the real object.

Guide a discussion of instances when it is important for students to estimate size and shape accurately (bagging their lunch, rearranging furniture in their room, putting games away, choosing food in a cafeteria line, etc.). Then elicit from the students careers in which spatial sense is particularly important (architecture, landscape artist, airplane pilot, fashion designer, engineer, cosmetologist, etc.).

An extension for this activity is to have students research specific careers and describe ways in which spatial sense is used. An alternate approach is to investigate highly paid careers and try to determine any relationships between spatial sense and success in these careers.

Activity 7: Fractional Beat

Standard

Fractions and Decimals (this activity focuses on common fractions)

Content connection

Music

Materials

For each small group of students: copy of a familiar song written in $\frac{4}{4}$ time; four 1" x 8" strips of construction paper (different colors); scissors; ruler; glue or glue stick; length of butcher paper or newsprint.

Distribute copies of the song selected. Introduce the song and sing it with your students. As a group, sing the song again, clapping the rhythm.

Have the students identify the time signature ($\frac{4}{4}$) and review the meaning of $\frac{4}{4}$ time (4 beats to a measure; each beat receives 1 count). Then guide the students to examine the measures and identify the different combinations of notes and rests used in composing measures (one whole note; two half-notes; one half-note, a quarter-note, and a quarter-rest; etc.).

Assign students to small groups and have them construct a set of fraction strips using four colors of 1" x 8" construction paper according to the following steps (construction paper colors for the entire class should be the same per step):

1. Use one full strip to represent 1 whole note or rest.
2. Cut a second strip into two 1" x 4" pieces to represent 2 half-notes or rests.
3. Cut a third strip into four 1" x 2" pieces to represent 4 quarter-notes or rests.
4. Cut a fourth strip into eight 1" x 1" pieces to represent 8 eighth-notes or rests.

Assign each group a line of the song and have them represent the rhythm with fraction strips. Have them glue their representation on a class sheet of butcher paper.

Add the title of the song, treble clef, measure bars, etc., and words, if desired, and display the song on the bulletin board.

Extend the lesson through the following suggestions:

1. Have students find other ways to use fractions in describing the song. (Each line is $\frac{1}{4}$ of the lines in a stanza, etc.)

2. Have students represent the rhythm for a "mystery" song with fraction strips; ask other students to clap the rhythm and try to guess the song.

3. Have students create their own measures (or songs), use fraction strips to represent the rhythm, and play the rhythm with rhythm instruments.

Activity 8: Wallpaper Patterns

Standard

Patterns and Relationships (this activity focuses on patterns)

Content connection

Art

Objective

Students will use their knowledge of tessellations to create their own wallpaper designs.

Materials

For the class: several discarded wallpaper books or samples (available from wallpaper stores); several containers of glue or several glue sticks; crayons, markers, colored pencils, or other available coloring materials. For each student: scissors, art paper.

This activity works well after students have had experiences with tessellations—working with pattern blocks and/or creating their own tessellations with cut paper. (See Giganti & Cittadino [1990] for step-by-step instructions on how to help students create their own tessellations.)

Distribute wallpaper samples (simple, obvious geometric patterns work best for initiating this activity) and ask students to outline the basic shape or shapes used by the creator of the design. Ask them to cut out the shapes and several repetitions of them from the wallpaper. Have students explore how the designer used the shapes to tessellate a plane (cover the available area with no omissions or overlaps).

Next, have students create a shape that will tessellate. It may be simple, such as a square or other rectangle, or as complex as they wish to make it. Have them create their own wallpaper samples using the tessellation they chose. These samples can be displayed or combined into a class book of wallpaper samples.

Closing Comments

The activities presented are in no way intended to be prescriptive; the authors have used them to help elementary school classroom teachers, both preservice and inservice, begin to develop their own curriculum connections for their classrooms. Most of the activities included provide opportunities not only for applying mathematics concepts but also a context and a reason for learning them.

References

Giganti, P., Jr., & Cittadino, M. J. (1990). The art of tessellation. *Arithmetic Teacher*, *37*(7), 6-16.

House, P. A. (1990). Mathematical connections: A long-overdue standard. *School Science and Mathematics*, *90*(6), 517-527.

Moore, E. H. (1902). On the foundations of mathematics. In J. K. Bidwell & R. G. Clason (Eds.). (1970). *Readings in the history of mathematics education*. Washington, DC: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Pearson, D., & Johnson, D. (1978). *Teaching reading comprehension*. New York: Holt.

Rider, C. (1990). Kentucky logic. *Kentucky Journal for Teachers of Mathematics*, Summer/Fall, 26.

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The authors appreciate the many teachers in classes and workshops who have used these activities with their students, given suggestions for improving them, and created their own. They are particularly indebted to Cindy Rider, classroom teacher at Alvaton Elementary School, Alvaton, KY, who enjoys integrating mathematics and Kentucky studies (see "Kentucky Logic" activity), and Marilyn Anderson, classroom teacher at T. C. Cherry Elementary School in Bowling Green, KY, who shared the basic idea for Fractional Beat.

PROJECT 2061 UPDATE

Scientific, Mathematical, and Technological Literacy for a Changing Future

American Association for the Advancement of Science

Project 2061 is a long-term reform intiative of AAAS to transform the nation's school system for the 21st century so that ALL students achieve literacy. To fulfill this vision of the future, the Project is developing a coordinated set of tools for a network of reformers.

Its 1989 report, *Science for All Americans*, outlined what ALL high school graduates should know and be able to do in *science*, *mathematics*, and *technology*. Since then, the project has been working with cross-grade/cross-discipline teams of educators at six demographically distinct sites across the country to design tools for school districts to use in developing their own curricula to achieve the SFAA goals. These tools include: *benchmarks for science literacy* at Grades 2, 5, 8, and 12; *curriculum models* with alternative approaches to teaching and learning; *blueprints for systemic reform* on related school issues; and a *curriculum-design system* linking Project 2061 tools to extensive resources.

Project 2061's Vision

- A FUTURE where ALL children can learn within a dynamic system created by teachers.
- LONG-TERM, SYSTEMIC REFORM of ALL aspects of K-12 education.
- SCIENTIFIC, MATHEMATICAL, AND TECHNOLOGICAL LITERACY as the common core of learning for citizens of the 21st century.
- CURRICULUM CONNECTIONS among natural and social science, mathematics, and technology and with the arts and humanities.
- LESS IS MORE when depth of useful learning prevails over sheer coverage of material, isolated facts, or memorization of technical vocabulary.
- LEARNING EXPERIENCES that engage students intellectually in activities relevant to their daily lives and their futures.
- DIVERSITY OF MATERIALS & METHODS in response to a diverse student population.
- TEACHERS AS PROFESSIONALS collaborating creatively in cross-grade/cross-disciplinary teams.

Project 2061 tools began to appear in 1993 and are being revised based on feedback from educators and classroom use. For information, write:

PROJECT 2061/AAAS
1333 H St. NW
Washington, DC 20005
Or call: 202/326-6666
FAX: 202/842-5196

Q & A

Benchmarks for Science Literacy
Project 2061

What are the Project 2061 benchmarks?

The Project 2061 benchmarks are statements of what *all* students should know or be able to do in science, mathematics, and technology by the end of Grades 2, 5, 8, and 12. The grade demarcations suggest reasonable progress toward the science literacy goals expressed in *Science for All Americans (SFAA)*. The Project 2061 *Benchmarks for Science Literacy* are one of a coordinated set of reform tools designed to help educators achieve the goals of *SFAA*.

What purposes are benchmarks intended to serve?

Benchmarks are a tool for curriculum design. They are **not** a curriculum nor an outline of a curriculum. Their topical organization follows the chapter headings in *SFAA*. As literacy goals, however, they are specific. Nevertheless, individual benchmarks should not be taught in isolation nor be turned into isolated test questions. Instead, they constitute a pool of specific learning outcomes from which educators can assemble conceptually interesting sets for organizing instruction and assessment.

Within Project 2061, benchmarks will play an important role in the design of curriculum blocks (courses, seminars, projects, and other instructional elements) and alternative curriculum models. Curriculum blocks will describe various contexts and activities for targeting multiple benchmarks; curriculum models will show how configurations of such blocks can create alternative K-12 curricula that achieve SFAA goals and meet Project 2061 standards.

How were benchmarks developed?

Benchmarks emerged from the R&D work over more than three years by the six Project 2061 site teams of K-12 educators. Their charge was to rethink curriculum in the light of *Science for All Americans*. The teams included elementary, middle, and high school teachers for science, mathematics, technology, and social studies, as well as principals and curriculum specialists. The teams were backed up by consultants in developmental learning and Project 2061 staff.

Each team independently concluded that a set of intermediary learning outcomes would be needed to guide their design of curriculum models. When teams later found that their grade-level expectations—though not their curriculum designs—were unexpectedly alike, they agreed to formulate a single set of science literacy goals or standards for what students should know and be able to do at different grade levels. These standards came to be called benchmarks and became one of the growing set of Project 2061 reform tools for achieving *SFAA* goals. The consequent collaborative effort to refine them resulted in the benchmark draft now being reviewed nationally.

Why Grades 2, 5, 8, and 12 instead of 4, 8, 12?

The Project 2061 teams were aware of the precedent of the 4, 8, and 12 sequence, however, each team came to the same conclusion: a single checkpoint between kindergarten and grade 8 would insufficiently guide those who design curriculum and instruction for young children. They argued that significant developmental and social reasons exist for distinguishing between the early and upper elementary grades. Explicit Grade 2 benchmarks may help prevent instruction being aimed prematurely at goals more suitable in Grades 4 or 5.

Why do benchmarks emphasize knowledge rather than skills?

They don't. They include calculator, computer, estimation, observation, manipulation, communications, and critical-thinking skills. But they also include knowledge—knowledge for making sense out of the world, knowledge as a foundation for life-long learning, and knowledge for problem solving and decision making. And like *SFAA*, the *Benchmarks for Science Literacy* will also emphasize the importance of knowing how science works.

Science literate adults should not be expected to *do* science in everyday life (unless, of course, their occupation is scientific) any more than they should be expected to compose music or try court cases. But just as they can appreciate harmony or argue law, they ought to be able to follow scientific news and weigh claims critically. Nevertheless, Project 2061 strongly believes that students should actively participate in scientific investigations rather than just read about how science works. Hands-on experience is a powerful boost to productive learning. A student who has never had to make and record accurate observations, control variables in an experiment, analyze a data set, or figure out how to test an hypothesis will find it difficult to understand what constitutes good science.

How do benchmarks relate to national standards?

A complete draft from the National Research Council's (NRC) standards for school science are now available; comparisons can now be made. Project 2061 staff believe that the NRC working committes share the Project 2061 commitment to science literacy for all Americans and the need to teach less than is in the current science curriculum so that what is taught can be learned well. Project 2061 and the NRC committee appear to be in agreement on the general outlines of what is important to learn. SFAA and the draft benchmarks continue to serve as inputs to the NRC effort.

How do benchmarks relate to assessment?

Assessment, like good instruction, also requires students to use knowledge in meaningful contexts. Benchmarks specify the knowledge and skills that students should be able to use by the end of grades 2, 5, 8, and 12. Thus, students should be assessed on their ability to draw on what they have learned to explain phenomena, think critically, and make informed decisions. Using benchmarks as isolated text questions is not compatible with such purposes. Instead, assessment should employ contexts that cut across a variety of benchmarks, requiring the student to apply his or her knowledge and thinking skills in novel settings. This contextual approach argues against framing each desired learning outcome in specific performance language, which might limit, even trivialize, the assessment of student progress.

How are the draft benchmarks being reviewed?

The draft benchmarks were reviewed by individuals and groups all over the United States and in several other countries as well. Education associations, scientific societies, state and national projects, government agencies, and other organizations interested in education reform identified reviewers.

In addition, more than a hundred different groups of reviewers were formed to represent the full range of state and school district demographics in America. Groups were generally comprised of 20 or more persons (teachers across the entire K-12 grade span and the full spectrum of science, mathematics, and technology subject matter, as well as curriculum specialists, administrators, scientists, and policymakers).

The reviewers were asked to address these questions: Is the science still right? Are the grade-level placements reasonable? Is the language clear and appropriate? Is the format helpful for its intended purposes? Each group submitted a single report reflecting the consensus reached after its members met to discuss and debate the draft recommendations.

When will "Benchmarks for Science Literacy" be available and how can it be obtained?

In response to feedback from the national review of the current draft benchmarks, Project 2061 completed the revision last summer and printed *Benchmarks for Science Literacy* this Fall. Its availability has been announced in AAAS publications (*2061 TODAY*, *Science Education News*, *Science*) and the professional journals of appropriate education, science, mathematics, and engineering associations. An announcement was also mailed to school districts. Use of the benchmarks will then be closely monitored and revisions made as necessary in the light of experience and of new research findings. **To order a copy of the Benchmarks, call Oxford University Press at 1-800-451-7556.**

How do benchmarks fit in the Project 2061 reform strategy?

Science for All Americans and *Benchmarks for Science Literacy* are the first of several curriculum-design tools being developed by Project 2061. Rather than create a curriculum for others to adopt, Project 2061 intends to foster curriculum diversity and local creativity by designing additional tools for educators to use in their own curriculum development. These reform tools will include the following:

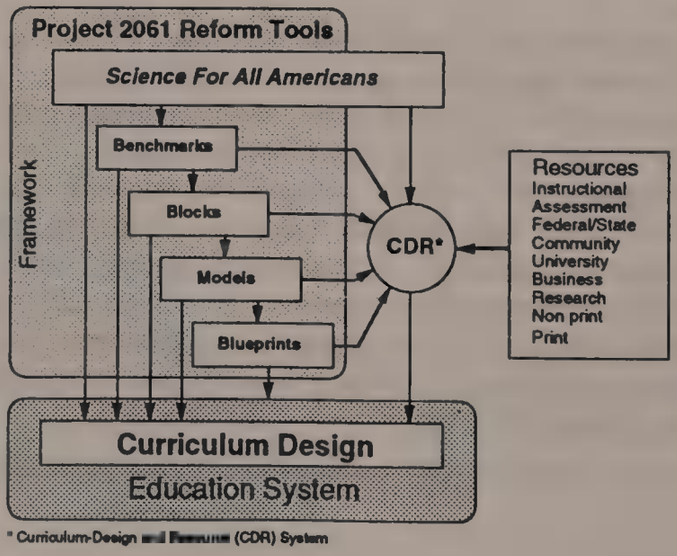
Designs for Science Literacy, a framework for conceptualizing curricula that meet *SFAA* and *Benchmarks* standards, will set out design principles for configuring such curricula and will present some alternative curriculum models to illustrate possibilities.

Curriculum blocks (courses, seminars, projects, and other instructional elements in an expanding collection) will target groups of benchmarks and, when assembled in various ways, will form alternative models of K-12 curricula.

Blueprints for Systemic Reform will describe how various aspects of the educational system—assessment, teacher education, school organization, etc.—need to change in order to accommodate new curricula successfully and will suggest workable strategies for achieving systemic reform.

A computerized Curriculum-Design and Resource (CDR) System will incorporate the framework, benchmarks, models, blocks, and blueprints and connect them to a variety of learning materials and other resources.

Project 2061 Vision of Systemic Reform



SSMILes

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Assembly Activities: Symmetries and Quasicrystal Patterns

Grade Level: 6-12

Mathematics Skills/Concepts
Creating designs and patterns with translational, rotational, and/or reflectional symmetries; problem solving.

Science Concepts/Skills
Understanding the geometry that underlies crystal structure, assembling quasicrystal patterns, problem solving.

Prerequisite vocabulary, information, and skills
Angle and angle measure in degrees including the fact that there are 360 degrees in a complete revolution, acute angle, obtuse angle, circle and interior of a circle, rhombus, edge, vertex, coincide, coincidence. For Procedure 7 only: isosceles triangle, ratio, proportion, using similar triangles to obtain proportions, solving proportions, and solving quadratic equations. Procedure 7 can be omitted.

Objectives
Students will be able to assemble periodic patterns, assemble patterns with reflectional symmetry, assemble patterns with five- and ten-fold rotational symmetry, assemble patterns using the Penrose edge-matching requirement, and assemble patterns using both the Penrose edge matching and vertex matching conditions.

Rationale
These activities supplement Clason, Peters, and Frank, 1993, p. 344-350 which contains a more detailed explanation of Penrose patterns and their relation to quasicrystals. Briefly, Penrose patterns were first investigated by Roger Penrose in the 1970s. Since then, they have been offered as a mathematical model for quasicrystal structure, a scientific discovery of the 1980s. Quasicrystals are materials displaying five-fold symmetries in three dimensions. Penrose patterns are also used in architecture, art, book cover designs, and quilt patterns.

Creating designs and tile patterns with the Penrose rhombs exposes students to a variety of mathematical ideas. The activities here give hands-on experience with symmetry: translational symmetry of periodic tile patterns, reflectional symmetry of various designs and tile patterns, and rotational symmetry, especially the unusual five-fold rotational symmetry found in Penrose patterns.

Lesson Outline

Time
Two or three 45-minute periods, including one full period for assembling Penrose patterns should be enough time, depending upon the amount of related exploration pursued.

Student Work Groups
These activities are suggested for use with groups of two or three students. Students should discuss and compare the geometric properties and aesthetic features of the designs and patterns they create. The teacher can circulate among the students to answer questions and discuss the patterns.

Materials
Each group will need five copies of the cutout sheet furnished with the article, scissors, and a flat surface (table top or floor). Students will cut out all rhombuses which are also called "Penrose rhombs" or tiles. The rhombs will be plain on one side, as shown in Figure 1, and marked on the other side, as shown in Figure 2.

Copying or gluing the sheets onto a material such as tag board, which is stiffer than paper but thin enough for easy cutting, will reduce the

Figure 1

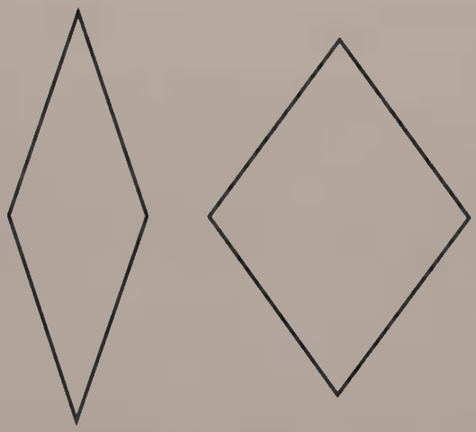
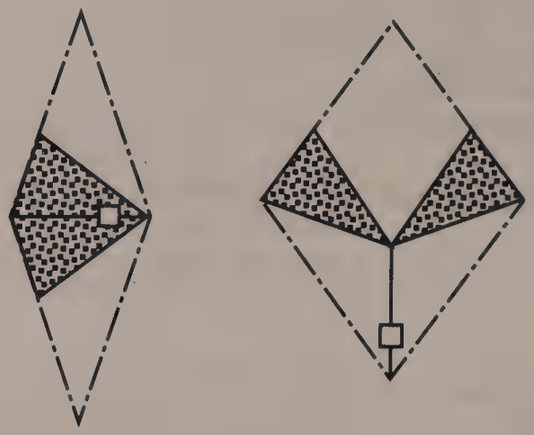


Figure 2



tendency of the tiles to override one another when tile patterns are assembled. The tag board rhombs could also be used as templates for making further unmarked rhombs which could be used to make nonPenrose designs. For Procedure 8, each group will also need a piece of poster board marked with a circle with a diameter roughly 10 times the edge length of a rhomb. The teacher could prepare this using a felttip pen or students could draw the circle.

Optional materials for teacher demonstration of symmetries

For younger students, the teacher may wish to review some basic geometric vocabulary and concepts in a preliminary discussion. Use a transparency of the rhombs shown in Figure 1 with the vertices of the thin rhomb labeled A, B, C, and D, and the vertices of the wide rhomb labeled E, F, G, and H. Ask the students to name all of the edges of each rhomb, and to name all of the acute angles and obtuse angles. Use double transparencies of the rhombs to demonstrate rotation. Place two identical copies of a rhomb, one above the other, so that the rhombs coincide. Holding the center point fixed, rotate the top copy through 90, 180, 270 and 360 degrees. Ask students to tell what the angle of rotation is in each position. To illustrate rotational symmetry, have the students identify the smallest angle of rotation for which the top and bottom rhombs coincide. Ask students to determine how many times this rotation would need to be repeated to make a total rotation of 360 degrees. To illustrate the reflectional symmetry, flip the top transparency and show that the flipped version can still be made to coincide with the bottom rhomb. If Miras or Reflectas are available, have students use them to determine lines of reflectional symmetry. A set of commercial pattern blocks could be used for activities paralleling those of Procedures 1-3, but obtaining patterns with 2-, 3-, 4, 6-, and 12-fold rotational symmetry and with reflectional symmetries in 2, 3, 4, 6, or 12 lines. These activities would be especially useful for students in grades 6-8. Students in grades 9-12 might also work on these activities before attempting Activity 8 since they provide experience that could prove useful with Activity 8. Activity 7 is intended only for students with some background in formal geometry. Rather than working on Activity 7, younger students might enjoy using tagboard templates of the rhombs and felt-tipped pens to create colorful posters for classroom display. Activity 8 was completed and enjoyed by the mathematically talented 7th, 8th, and 9th graders in one of the author's classes, but some students in regular classes at this level may find the puzzle frustrating.

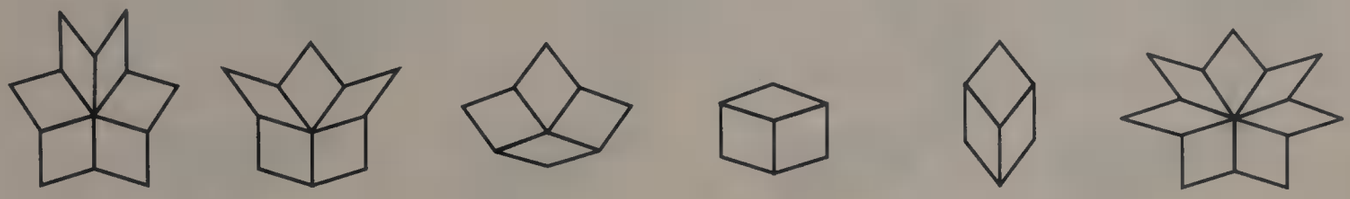
Teacher Note

Before starting students on Procedures 7 and 8, you may wish to try some patterns yourself. The Penrose puzzle is tricky, and experience recognizing situations that require backtracking will help when students ask for assistance. See Clason, Peters, and Frank, 1993, pp. 344-350 for additional information.

Evaluation

Patterns assembled by groups can be informally evaluated for required symmetries and assembly as the students complete each task—for example, the teacher might suggest the placement of a rhomb to

Figure 3



two 36 four 72	two 36 two 108 one 72	three 72 one 144	one 144 two 108	one 72 two 144	four 36 three 72

maintain reflectional symmetry. Understanding of symmetries can be evaluated by furnishing sheets with a variety of designs and patterns and asking students to draw all lines of reflectional symmetry, to report what "fold" any rotational symmetries have (e. g., 3-fold, 5-fold, etc.), and to tell whether or not each pattern is periodic. Students might also keep a journal including photographs of the patterns they create with a discussion of their symmetries.

Procedure

The procedures are written so that they can be copied and distributed to students. Answers to some of the activity questions follow:

Activities 2 and 3

Figure A has five lines of reflectional symmetry. The wide and thin rhombs have two lines of reflectional symmetry each. Each rhomb has two-fold rotational symmetry. A design with ten-fold rotational symmetry and reflectional symmetry in ten lines can be made by placing ten thin rhombs around a central point. Further examples of designs are shown in Figure 3, with the angle measure for each for the angles which meet at the center and sum to 360 degrees. They all have reflectional symmetry in one line. Incidentally, the six arrangements of Figure 3 together with the arrangement shown in Figure A of Activity 1 are the only seven arrangements of the rhombs around a point that can appear in a Penrose pattern.

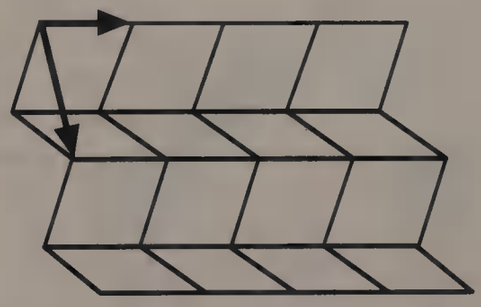
Activity 4

Wide rhomb's angles in degrees: 108 (obtuse) and 72 (acute); for the thin rhomb, 144 and 36, respectively.

Activity 5

Arrows for translations (other answers are possible):

Figure 4



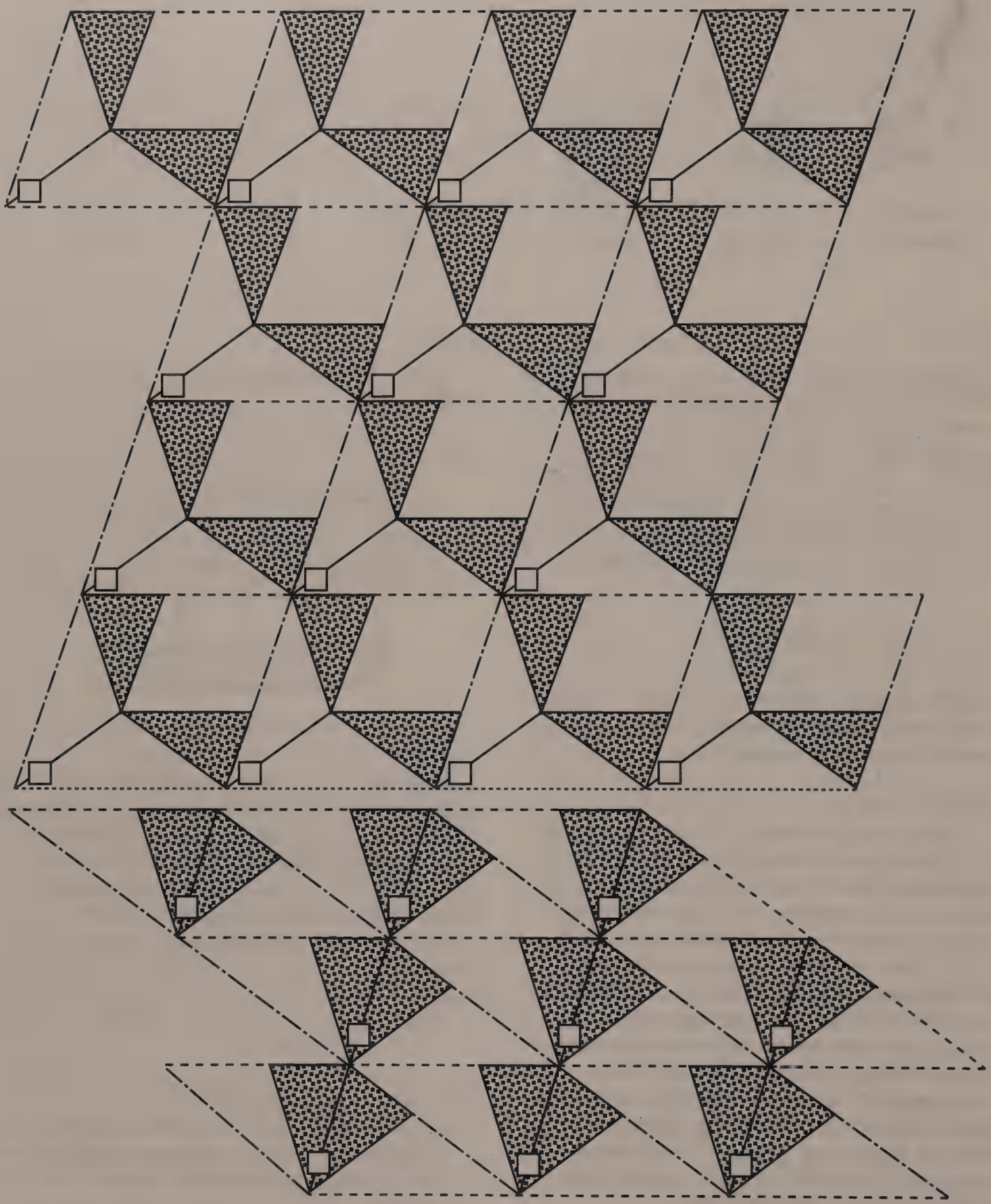
Activity 7

Segments AC, DC, and BD have the same length. ΔABC and ΔBCD are each isosceles. ΔABC and ΔDCA are similar. These similar triangles yield the proportion $x/1 = (1 - x)/x$ so that $x2 = 1 - x$ or $x2 + x - 1 = 0$. Using the quadratic formula and discarding the negative root, $x = (-1 + \sqrt{5})/2$ or about .618, the golden ratio. (Some sources call the reciprocal, about 1.618, the golden ratio.)

Reference

Clason, R. G., Peters, W., and Frank, M. (1993). Quasicrystals and the Penrose Patterns: A Geometric Activity with a Scientific Application. *School Science and Mathematics*, *93*, (7), 344-45.

Note: Dyanne Tracy's address is Oakland University, 502 O'Dowd Hall, Rochester, MI 48309-4401 Penrose tiles are produced by and available from Kadon Enterprises, Inc., 1227 Lorene Drive, Suite 16, Pasadena, MD 21122, (410)437-2163. Under license from Pentaplex LTD UK.



Student Activities: Symmetries and Quasicrystal Patterns

Use the scissors to cut all of the diamond shaped pieces (also called Penrose rhombs) from the sheets you have been provided. Cut on the dotted lines. When all the rhombs have been cut out, give each person 10 wide rhombs and 10 thin rhombs. Turn the rhombs so that the BLANK side is facing up.

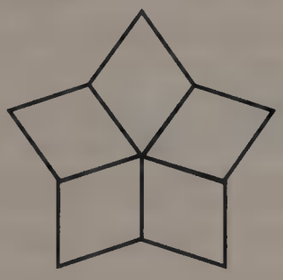
Activity 1: Making Designs

Make a design using some or all of your rhombs. Your design should have no gaps. It should also be edge-to-edge, that is, a vertex of one rhomb should not touch the edge of another rhomb except at another vertex. Try some more designs.

Activity 2: Rotational Symmetry

Figure A has rotational symmetry because if a transparent copy were placed on top it and then rotated about the center, it would match the original with a rotational angle of less than 360°. The design has five-fold rotational symmetry since if your rotate it through the smallest angle possible for matching and repeat this five times, the design will be in its original position. Do the designs made by anyone in your group have rotational symmetry? Do the wide and thin rhombs themselves have rotational symmetry? Make a design with two-fold rotation symmetry. Make a design with ten-fold rotational symmetry.

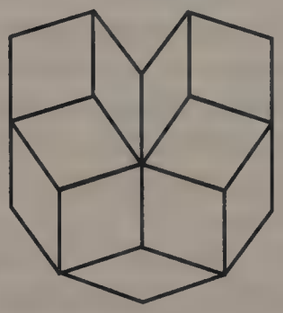
Figure A



Activity 3: Reflectional Symmetry

Figure B has reflectional symmetry because if it were folded on a vertical line through the middle, called a line of reflection, the two half patterns would coincide. How many lines of reflection does Figure A have? Make two different designs that have reflectional symmetry and compare them with the designs of others in your group. How many lines of reflectional symmetry does each design have? Do the wide and thin rhombs themselves have reflectional or rotational symmetry?

Figure B



Activity 4: Problem

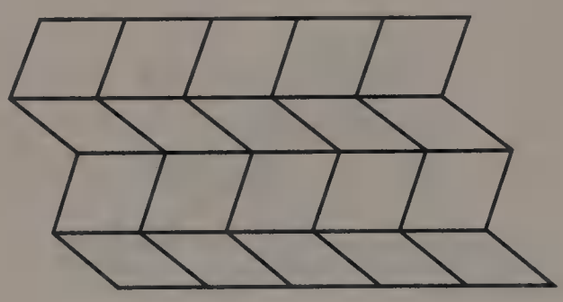
Work with your group members to find the measure of each angle of a wide rhomb and a thin rhomb, without using a protractor. HINT: Looking at some designs might be helpful. Write a sentence or two telling how you solved this problem.

Wide rhomb's obtuse angle: _____
Wide rhomb's acute angle: _____
Thin rhomb's obtuse angle: _____
Thin rhomb's acute angle: _____

Activity 5: Periodic Patterns—Translational Symmetry

Figure C shows a periodic tile pattern because it could be extended maintaining the same pattern to cover the entire plane. A tile pattern has translational symmetry if a transparent copy placed over it in a matching position could be slid in some direction to another matching position. A periodic tile pattern has translational symmetries in at least two directions. Find two directions in which the pattern of Figure C has translational symmetry and mark each of them with an arrow.

Figure C



Make a different design that could be extended to a periodic tile pattern. Compare the designs. Do any of them have reflectional symmetry? 2-fold rotational symmetry? 5-fold rotational symmetry?

Activity 6: Edge Matching Conditions

Turn your tiles over so that blank sides are down and the patterned sides up. Make designs using the edge matching condition: Place tile edges so that the shaded sections match to form smaller, thin rhombs and the unshaded sections form smaller, wide rhombs. (Ignore the small square near one vertex for this activity.) Make designs that have rotational symmetry and designs that have reflectional symmetry.

Work with the others in your group and use at least 30 wide rhombs and 20 thin ones to make a large design observing the edge matching condition. If your design is periodic, make another that has five-fold rotational symmetry. If your design is not periodic, make another that is periodic.

Activity 7: The Relation Between the Small and Large Rhombs

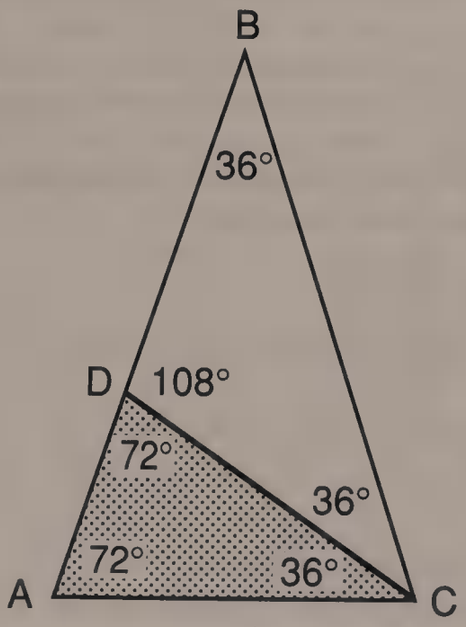
Figure D shows half of a large thin rhomb. The angle measures should include the ones you found in Procedure 4.

How are AC, DC, and BD related?

What kind of triangles are ΔABC and ΔBCD?

What is the relationship between ΔABC and ΔDCA?

Figure D



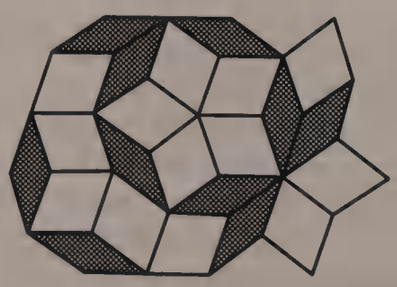
Problem: Find the ratio of the length of AC to the length of BC. Note that this is the ratio of the diagonal of the large thin rhomb to its edge length, and also the ratio of the edge length of the small thin rhomb to the edge length of the large thin rhomb. HINT: Let the length of BC be one unit and the length of AC be x units. Then the problem is to find $x/1$ or x. The value of x is called the golden ratio.

Activity 8. Vertex Matching Condition and Edge Matching Condition

This is the Penrose-quasicrystal puzzle. Roger Penrose created this puzzle in the 1970s, partly to cheer up a sick friend in the hospital! In the 1980s, the pattern was suggested as a two-dimensional model for the three-dimensional structure of materials called "quasicrystals."

The objective of the puzzle is to assemble enough of a Penrose pattern to cover the entire circle and its interior completely. Tiles may extend partially beyond the circle. There are two matching rules for the puzzle: the edge matching rule of Procedure 6—the rhombs must be placed so that the shaded sections match to form smaller, thin rhombs and the unshaded sections form smaller, wide rhombs— and a vertex matching rule which involves the small square found in a corner of each tile. The vertex matching rule says that tiles must be placed so that where vertices meet at a point, all the vertices have a small square, or none of the vertices meeting the point have a square. Work as a group and assemble your pattern on the piece of poster board with the large circle. HINTS: Start at the center and work outward. There are no periodic solutions, but there are solutions with five-fold rotational symmetry and there are solutions with reflectional symmetry. If you get to a point which seems impossible, it may be necessary to remove a large part of your pattern and start again.

Figure E



BOOK REVIEWS

S. Wali Abdi, Editor

Numbers and Functions: Steps Into Analysis

Author
R.P. Burn
Cambridge University Press
40 West 20th Street
New York, New York 10011-4211
First Published 1992
First Paperback Edition (with corrections) 1993;
328 pages
$24.95

Reviewer
Bob Prielipp
University of Wisconsin-Oshkosh
Oshkosh, WI 54901-8631

Do the following questions interest you?

1. If $\sum a_n^2$ is convergent and $a_n \geq 0$, is $\sum a_n/n$ convergent?
2. Is there a function f: $(0,1] \rightarrow R$ (where R is the set of real numbers) which is discontinuous at each rational point of (0,1] and continuous at every irrational point of (0,1]?
3. Can it be the case that

$$\lim_{x \rightarrow a^+}\left(\frac{f(x)}{g(x)}\right) \text{ exists, but } \lim_{x \rightarrow a^+}\left(\frac{f'(x)}{g'(x)}\right)$$

(where $f'(x)$ denotes the derivative of $f(x)$ with respect to x) does not exist even though both f and g are differentiable functions of the variable x?

If your answer is "yes," then you will be enthralled by *Numbers and Functions: Steps Into Analysis.*

The author, R.P Burn, has written a book whose goal is to bridge the gap between a nontheoretical beginning course in calculus and a class in advanced calculus or introductory real analysis. He tries to accomplish this by presenting the usual definitions, giving some examples, and then raising a number of fairly detailed, thought-provoking questions. Any reader who works through a substantial number of the given questions would enhance considerably her/his mathematical knowledge. Since fairly complete answers are given to almost all of the questions raised, even the person who just reads the book and checks out the details of an answer would probably benefit. Only a very small number of misprints/errors were found and the vast majority of these will be easily recognized.

Numbers and Functions is divided into two sections: Part I, "Numbers," and Part II, "Functions." Part I includes chapters on counting numbers and mathematical induction; order; sequences; completeness; and series. Part II reviews functions and continuity, neighborhoods, limits of functions; continuity and completeness; derivatives (tangents); differentiation and completeness; integration; indices and circle functions; and sequences of functions. Each chapter consists of exposition, questions, a summary, an historical note, and answers. The book also includes an appendix on geometry and intuition and an appendix which contains eight multiple-choice questions for student discussion and seventeen problems for group or individual investigation.

I believe that a serious student of mathematics (and even some teachers of mathematics) would find conscientious study of this book of considerable value.

History of Women in Science for Young People

Author
Vivian Sheldon Epstein
(Illustrated by: Vivian Sheldon Epstein)
VSE Publisher
212 South Dexter
Denver, CO 80222
1994
Soft cover $7.95; Hardback $14.95
ISBN: 0-9601002-7-X

Reviewer
Paula G. Wilder, Executive Director
Discovery Works . . . a children's museum
Blacksburg, VA 24060-8186

Thirty brief biographies of women scientists appear in this book, ranging from Hypatia, ancient

Roman mathematician, astronomer, and philosopher, and Hildegard, a Middles Ages nun who established many of the principles of modern medicine to Lise Meitner, a 19th century Austrian scientist who was the first researcher to understand the process of splitting atoms and Myra Adele Logan, 20th century American surgeon, one of the first physicians to operate on the human heart. In addition, the *History of Women in Science for Young People* provides a chronology of more than 150 other women who contributed significantly to the sciences. Epstein's concise biographies feature the specific accomplishments of each woman, often tracing her career as a scientist to childhood interests. Epstein furnishes a backdrop created from her subjects' beliefs and families and, so, successfully makes these women come alive.

By selecting women from a variety of fields including environmental science, chemistry, physics, aeronautics, medical science, psychology, computer sciences, genetics, nuclear science, industrial medicine, archaeology, marine botany and anthropology, she not only broadens a reader's view of who can do science, she also broadens perspectives of how many forms science can take. By focusing her biographical profiles on problem finding and problem solving, she develops the notion that science is a fun, challenging, life-long inquiry process rather than just isolated, Nobel Prize-winning discoveries.

This *History* is particularly suited to teachers educated in an era when women's accomplishments were not found in textbooks. Its 70-item bibliography provides an excellence reference for students and teachers alike. The type is large enough to make it accessible to the intermediate grade student. Its inexpensive 40-page paperback format is a welcome addition to any classroom library designed to stimulate children's interest in science careers and should be part of all school collections. In addition, the format puts the thirty main biographies on separate pages, adding to a teacher's convenience. In future editions, actual photographs of these women (when possible) would be a helpful improvement. There is evidence that the more our young girls see real women in these roles, the more likely they will be to aspire to science careers themselves.

School and classroom libraries should have this helpful reference as much for focused study as for browsing. It will inspire all young people — boys and girls alike.

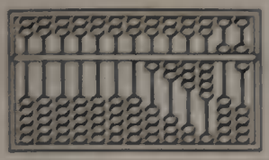
A Mathematical Pandora's Box

Author
Brian Bolt
Cambridge University Press
40 West 20th Street
New York, NY 10011-4211
1993; 126 pages
Paperback

Reviewer
Mary T. Whalen
University of Wisconsin
Stevens Point, WI 54481

This delightful book is a collection of 142 mathematical puzzles, games, and tricks. Some will be familiar to teachers and others are brand new. An example of a puzzle which is included: Use 18 standard dominoes to form a 6 x 6 square in such a way that the number of dots in each of the six rows and columns and the two main diagonals is the same. This and the other 141 activities involve some good problem solving and could be assigned for individual or cooperative work. Some of the activities would make excellent choices for a "Problem of the Day." The problems vary in difficulty; I think teachers in Grades 5 to 12 would be able to find material suitable for their classes. The book was published in England and so there are some English spellings and terms which would not be familiar to American students but could be easily changed by the teacher. An example is the use of pounds for money, instead of dollars. Answers to all problems are provided at the back of the book and are printed in red, which makes them easy to locate. I highly recommend this book for middle school/ high school mathematics teachers.

Note: S. Wali Abdi's address is The University of Memphis, Department of Curriculum and Instruction Leadership, 401 A Ball Hall, Memphis, TN 38152.



PROBLEMS

Richard A. Gibbs and László Szücs, Editors

This department offers readers an opportunity to exchange interesting mathematical problems and solutions. Proposals are always welcomed. Please observe the following guidelines when submitting solutions or proposals:

1. Solutions and proposals should appear on separate sheets, each indicating the name of the sender. Drawings must be suitable for reproduction. Proposals should be accompanied by solutions. An asterisk (*) indicates that neither the proposer nor the editors have supplied a solution.

2. The editors encourage undergraduate and pre-college students to submit solutions. Teachers can help by assisting their students in submitting solutions. Student solutions should include the class and school name.

3. Send submittals to **R.A. Gibbs, SSM Problem Department, Fort Lewis College, Durango, CO 81301-3999**; or by Email to **gibbs_d@FLC.Colorado.EDU** (TEX source code accepted); or by FAX to **303-247-7434**.

PROBLEMS

Solutions should be mailed before February 28, 1995. (Solutions sent after that date will be considered as deadlines permit.)

4447: *Proposed by Murray S. Klamkin, Edmonton, AB, Canada.*

The signed perpendicular distances from the vertices of a triangle with angles A, B, C to a line in its plane are respectively equal to p, q, r. Determine the respective sides a, b, c of the triangle.

4450: *Proposed by John Winterink, Albuquerque, NM.*

By a *generalized circle* we mean a circle or a straight line. By a *Pythagorean circle* we mean a circle whose center has integer coordinates and whose radius is a positive integer.

(a) Determine eight generalized circles, each of which is tangent to each of the three given Pythagorean circles:

$$(x+3)^2+(y-3)^2=36, \quad (x-1)^2+(y+5)^2=4, \quad (x-2)^2+(y+2)^2=1.$$

(b*) Give an example of three Pythagorean circles such that there exist eight Pythagorean circles, each of which is tangent to each of the three given Pythagorean circles.

Editors' Note: Problems 4447 and 4450 appeared originally in the November, 1993 issue. Since no solutions were received, they remain "open" until February 28, 1995.

4488S: *Student Problem* – only undergraduate and pre-college students are eligible to submit a solution.

Proposed by William R. Klinger, Upland, IN.

Let $a_1 < a_2 < a_3$ be the real zeros of a cubic polynomial let c_1 and c_2 $(a_1 < c_1 < a_2 < c_2 < a_3)$ be the critical numbers whose existence is guaranteed by Rolle's Theorem. Prove that the arithmetic average of the squares of the three real zeros is greater than the product of the two critical numbers. (See Problem 459 in the September 1991 issue of the *College Mathematics Journal* for a related result.)

4489: *Proposed by Andrew Cusumano, Great Neck, NY.*

Let O be the center of equilateral $\triangle ABC$, and let M be the midpoint of side AB. Furthermore, let D and E be points of AO and BO, respectively, equidistant from O. Finally, let N be the intersection of MD and AE. Show that NO is parallel to AC.

4490: *Proposed by Roger Izard, Dallas, TX.*

One day, a young mathematician visited a palmist. The palmist examined his hand and then, while still holding his hand, said to him:

"If you get rich, either you will be a bachelor or you will become famous."

"If you live a life of obscurity, you will have a long life, and either you will not beome rich or you will get married."

"If your life is a short one, then either you will become rich or you will become famous."

"Wait a minute! I have been overlooking one important line. What I said previously may not be true, but in that case, you will never be famous, and if you have a long life, you will become rich."

Suddenly, the young man jumped up and said, "I refuse to pay my fee! You haven't said anything! All that you have said constitutes a tautology which is true under any circumstance."

Was the mathematician justified in his attitude?

4491: *Proposed by Richard L. Francis, Cape Girardeau, MO.*
Show that it is impossible, using compass and unmarked straightedge, to construct a triangle all of whose angles are of prime degree measure.

4492: *Proposed by Herta T. Freitag, Roanoke, VA.*
For $k = 3, 4, \ldots$ and $n = 1, 2, \ldots$, the *polygonal* (or *figurate*) *number* $P(k, n)$ is defined by

$$P(k, 1) = 1, \qquad P(k, n) = T_n + (k-3)T_{n-1} \text{ if } n \geq 2$$

where T_n is the nth triangular number $n(n+1)/2$.
For any $k \geq 3$, $n \geq 1$, and $m \geq 2$, find an expression for the determinant

$$D(k, n, m) = \begin{vmatrix} P(k,n) & P(k,n+1) & \ldots & P(k,n+m-1) \\ P(k+1,n) & P(k+1,n+1) & \ldots & P(k+1,n+m-1) \\ \vdots & \vdots & & \vdots \\ P(k+m-1,n) & P(k+m-1,n+1) & \ldots & P(k+m-1,n+m-1) \end{vmatrix}$$

4493: *Proposed by Murray S. Klamkin, Edmonton, AB, Canada.*
Characterize the set of all sets $\{A_1, A_2, \ldots, A_{2n}\}$ of points in Euclidean space such that, for any point P,

$$A_1P^2 + A_3P^2 + \cdots + A_{2n-1}P^2 = A_2P^2 + A_4P^2 + \cdots + A_{2n}P^2.$$

SOLUTIONS

The Editors will be happy to consider new solutions, generalizations, or other comments on published solutions.

4446S: *Student Problem* – only undergraduate and pre-college students are eligible to submit a solution.
Proposed by Darryl Egley, Kirksville, MO.
Let P be any point, and let a, b, c and d be the respective distances from P to the vertices of a rectangle $ABCD$ in 3-dimensional space. Prove that $a^2 + c^2 = b^2 + d^2$.

Solution by Edward Early and Sean Owen (independently), both 10th grade students at the Science Academy of Austin at LBJ, Austin, TX.
Let P_1 be the foot of the perpendicular from P to the plane of $ABCD$, let w, x, y, z be the distances from P_1 to sides AB, CD, AD, BC respectively, and let a_1, b_1, c_1, d_1 be the distances from P_1 to A, B, C, D respectively. Then, by the Pythagorean Theorem,

$$a_1^2 = w^2 + y^2, \quad b_1^2 = w^2 + z^2, \quad c_1^2 = x^2 + z^2, \quad d_1^2 = x^2 + y^2$$

so that $a_1^2 + c_1^2 = b_1^2 + d_1^2$.
Now, let h be the distance from P to the plane of $ABCD$, and let a, b, c, d be the distances from P to A, B, C, D respectively. Then, by the Pythagorean Theorem,

$$a^2 = h^2 + a_1^2, \quad b^2 = h^2 + b_1^2, \quad c^2 = h^2 + c_1^2, \quad d^2 = h^2 + d_1^2$$

so that $a^2 + c^2 = b^2 + d^2$.
Also solved by: the proposer.

4447: See the Problems section.

4448: *Proposed by Richard L. Francis, Cape Girardeau, MO.*
For any positive integer k, define the kth *century* to be the set $\{(k-1) \cdot 100 + 1, \ldots, k \cdot 100\}$.
(a) Show that the century beginning with $100! + 1$ does not contain any primes.
(b) Is the century beginning with $100! + 1$ the first primeless century?

Solution to a) by Trygve Breiteig, Kristiansand, Norway; Kou-Tsai Chang, Hsin-chu, Taiwan; Edward Early, a 10th grade student at the Science Academy of Austin at LBJ, Austin, TX; N.J. Kuenzi, Oshkosh, WI; Charles McCracken, Dayton, OH; Bob Prielipp, Oshkosh, WI; Heinz-Jürgen Seiffert, Berlin, Germany; and David C. Wilson, Eden, NC (independently).
Clearly, $k | 100! + k$ for $k = 2, 3, \ldots, 100$. Now, the well-known Wilson's Theorem states that if p is a prime, then $(p-1)! \equiv -1 \pmod{p}$. Since 101 is prime, by Wilson's Theorem, $101 | 100! + 1$.
Also solved by: Charles Ashbacher, Cedar Rapids, IA; and the proposer.

A composite solution to b) by N.J. Kuenzi, Oshkosh, WI and Heinz-Jürgen Seiffert, Berlin, Germany (independently).
We will show that the century beginning with $2 \cdot 98! + 1$ is also primeless. Clearly, $k | 2 \cdot 98! + k$ for $k = 2, 3, \ldots, 98$. Also, since $99 | 98!$ and $100 | 98!$, we see that $k | 2 \cdot 98! + k$ for $k = 99, 100$. From $2 \equiv -99 \equiv 100 \cdot 99 \pmod{101}$, and from $100! + 1 \equiv 0 \pmod{101}$ (by Wilson's Theorem), we see that $101 | 2 \cdot 98! + 1$. The result follows.
The fact that $2 \cdot 98! + 1 \equiv 0 \pmod{101}$ also follows from a generalization of Wilson's Theorem found on p. 131 of *An Introduction to Number Theory*, 3rd ed. by Calvin F. Long (Prentice Hall, 1987):
If p is prime and h, k are non-negative integers such that $h + k = p - 1$, then $h!k! \equiv (-1)^{k+1} \pmod{p}$.

For $p = 101$, $h = 2$ and $k = 98$, we have $2!98! \equiv (-1)^{99} \pmod{101}$.

Also solved by: Charles Ashbacher, Cedar Rapids, IA; Bob Prielipp, Oshkosh, WI; David C. Wilson, Eden, NC; and the proposer.

Ashbacher wrote a *UBASIC* program to find that the first primeless century begins with 1,671,801. Prielipp cited the same result in W. Sierpinsky's *A Selection of Problems in the Theory of Numbers* (Pergamon Press, Macmillan Co., 1964, p.43). The proposer noted that, though this century contains the square 1,671,849, it has no cubes or higher powers, no (odd or even) perfect numbers, no triangular numbers and no palindromes.

4449: *Proposed by Mango Ahuja, Cape Girardeau, MO.*

Given a triangle ABC and a square of side length x with one side on the line through B and C, and two vertices on the respective sides AB and CA. Show that $2x$ is the harmonic mean of a and h, where a is the length of side BC and h is the length of the altitude through A.

Solution by Kuo-Tsai Chang, Hsin-chu, Taiwan; Heather Lecceardone, a student at St. Bonaventure University, St. Bonaventure, NY; Joe Howard, Las Vegas, NM; and Fred A. Miller, Elkins, WV.

Since the harmonic mean of a and h is

$$\frac{2}{1/a + 1/h} = \frac{2ah}{a+h},$$

it suffices to show that $x = ah/(a+h)$.

Let the square be $DEFG$ with D on AB, G on AC. Since $\triangle ABC \sim \triangle ADG$, we have $(h-x)/x = h/a$. Therefore $x = ah/(a+h)$ and the result follows.

Also solved by: Sam Baethge, Austin, TX; and Paul M. Harms, Upland, IN.

There were eight partial solutions which considered only the case of a square *inscribed* in an acute angled triangle ABC.

4450: See the Problems section.

4451: *Proposed by Michael Brozinsky, West Hempstead, NY.*

Given polynomials $a(x) = \sum_{i=0}^{n} a_i x^i$ and $b(x) = \sum_{i=0}^{m} b_i x^i$ with $n \geq m$, let $q(x) = \sum_{i=0}^{n-m} q_i x^i$ and $r(x) = \sum_{i=0}^{m-1} r_i x^i$ be the quotient and remainder, respectively, obtained when $a(x)$ is divided by $b(x)$. Find an $n+1 \times n+1$ matrix $B = (\beta_{ij})$ such that

(*i*) the entries of B are constants or expressible in terms of $b_0, \ldots, b_m$,

(*ii*) $\beta_{ii} = 1$ for $i = n-m+2, \ldots, n+1$,

(*iii*) $\beta_{ij} = 0$ for $j = n-m+2, \ldots, n+1$, $i \neq j$, and

(*iv*) the system $BX = A$ of linear equations where

$$A = \begin{pmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_0 \end{pmatrix}$$

has the solution

$$X = \begin{pmatrix} q_{n-m} \\ \vdots \\ q_0 \\ r_{m-1} \\ \vdots \\ r_0 \end{pmatrix}$$

Example: Let $a(x) = 3x^3 + 5x^2 + 2x - 2$ and $b(x) = x^3 + x^2 + x + 1$. Then we have $q(x) = 3$, $r(x) = 2x^2 - x - 5$, and $BX = A$ becomes

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ -1 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 2 \\ -2 \end{pmatrix}$$

Solution by Heinz-Jürgen Seiffert, Berlin, Germany.

Define the $n+1 \times n+1$ matrix $B = (\beta_{ij})$ by:

$$\beta_{ij} = \begin{cases} b_{m-i+j}, & \text{if } \max(1, i-m) \leq j \leq i \leq n-m+1; \\ b_{m-i+j}, & \text{if } n-m+2 \leq i \leq n+1 \text{ and } \max(1, i-m) \leq j \leq n-m+1; \\ 1, & \text{if } n-m+2 \leq i = j \leq n+1; \\ 0, & \text{otherwise.} \end{cases}$$

It is easily seen that B satisfies (i), (ii) and (iii). Next, note that

$$b(x)q(x) = \sum_{i=0}^{n} \sum_{k=\max(0,i-n+m)}^{\min(i,m)} b_k q_{i-k} x^i \tag{1}$$

If $1 \le i \le n-m+1$, then $m \le n+1-i \le n$, and comparing coefficients using (1) yields

$$a_{n+1-i} = \sum_{k=\max(0,m+1-i)}^{m} b_k q_{n+1-i-k}.$$

Setting $j = k-m+i$ and noting that $\max(0, m+1-i)+i-m = \max(1, i-m)$, we see that

$$a_{n+1-i} = \sum_{j=\max(1,i-m)}^{i} b_{m-i+j} q_{n-m+1-j},$$

or, by the definition of B,

$$a_{n+1-i} = \sum_{j=1}^{n-m+1} \beta_{ij} q_{n-m+1-j} + \sum_{j=n-m+2}^{n+1} \beta_{ij} r_{n+1-j} \quad \text{for} \quad 1 \le i \le n-m+1. \tag{2}$$

Now, if $n-m+2 \le i \le n+1$, then $n+1-i \le m-1$, and again comparing coefficients using (1) yields

$$a_{n+1-i} = \sum_{k=\max(0,m+1-i)}^{n+1-i} b_k q_{n+1-i-k} + r_{n+1-i}.$$

Setting $j = k-m+i$ as above we obtain

$$a_{n+1-i} = \sum_{j=\max(1,i-m)}^{n-m+1} b_{m-i+j} q_{n-m+1-j} + r_{n+1-i},$$

or, by the definition of B,

$$a_{n+1-i} = \sum_{j=1}^{n-m+1} \beta_{ij} q_{n-m+1-j} + \sum_{j=n-m+2}^{n+1} \beta_{ij} r_{n+1-j} \quad \text{for} \quad n-m+2 \le i \le n+1. \tag{3}$$

Equations (2) and (3) verify that $A = BX$.

Also solved by: the proposer.

EDITORS' NOTES

Late solutions were received from Nirusha Ernest, Encino, CA for Problems 4380 and 4429, and Bob Prielipp, Oshkosh, WI for Problem 4405. Frank J. Flanigan, San Jose, CA, commented that the statement of Problem No. 4339S should have included the phrase "for all positive integers n". Stanley Rabinowitz, Westford, MA, sent a non-trigonometric, purely geometric solution to Problem No. 4356S. H. Don Allen, Iqaluit, NT, Canada, sent an alternate solution to Problem No. 4410S.

IN MEMORIAM

It was with sorrow that we learned of the passing of Harry Ruderman. Harry passed away in April. He was an active participant, both as a poser and a solver, in the *SSM* Problem Department for many years. His contributions will be missed.

We were also saddened to hear that Harry's wife Ruth passed away in June. They were both very involved in working on problem solving activities with students. They were long-time contributors to the New York State Mathematics League and the American Regions Mathematics League.

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